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Price level determination: Ricardian vs. non – Ricardian policies

by

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Major Professor

For the Major Program

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CHAPTER 1. INTRODUCTION

"Inflation is almost always a monetary phenomenon"

M. Friedman

The monetarist theory gives a clear definition of the actions and limitations of the monetary and fiscal authorities. According to that theory, the monetary authority should set the money growth rate with a clear objective of price level stability and it should be independent of the fiscal authority to achieve its goals. Whereas the fiscal authority will be responsible for its own house and should aim at keeping a balanced budget given the price level at all time. However, Sargent and Wallace (1981) and the advocates of the fiscal theory of the price level (FTPL) indicated that this distinction between the monetary and fiscal authorities as two different parts of the government is not as strict as it seems and their relationship has important consequences on the price level.

Almost two decades ago Sargent and Wallace (1981) showed that if monetary policy is defined as open market operations, then even in a monetarist environment the fiscal authority may act in a dominant fashion and exercise significant control over the inflation rate. That is, it is possible for the monetary authority to be a 'follower' and to lose control over the price level.

Advocates of the fiscal theory of the price level, such as Woodford (1995), Leeper (1991), Sims (1993) and Cochrane (1998) take this argument one step further and argue that it is actually possible for both the monetary and the fiscal authorities to be dominant such that, fiscal and monetary authority are not bound by the solvency of its budget constraint.

The fiscal theory of the price level (FTPL) arose as an attempt to explain the weak relation between monetary aggregates and the inflation rate in today's technologically advanced environment where, it is harder to control the monetary aggregates.

The theoretical possibility of non –Ricardian policies is discussed by many economists and there are various studies on the relationship between the fiscal debt and the price level. However, there are very few empirical studies testing the FTPL. The studies by Canzoneri, Cumby and Diba (1998) and Cochrane (1998) have conflicting results. These studies aim at analysing the behaviour of the fiscal authority through the government budget constraint. However, due to the restrictions of the models the relation between the government debt, government surplus and the price level is missed.

In this study we use structural vector autoregressive (VAR) and structural vector error correction (VECM) models to analyse the US price level in an IS –LM framework under the arguments of monetary theory, unpleasant monetarist arithmetic and the FTPL.

The study is organized as follows. Chapter Two presents a discussion on the Ricardian and non –Ricardian policies in the context of the monetarist theory, the unpleasant monetarist arithmetic and the FTPL. The theoretical and empirical studies on the FTPL are also reviewed in that chapter. Chapter Three introduces the VAR and VECM econometric models that will be used for the empirical part of the study. Chapter Four states the identification restrictions we propose, to distinguish monetary theory, unpleasant monetarist arithmetic and FTPL for both the structural VAR and structural VECM models. The properties of our data set and the theoretical discussions on the econometric tools we use for cointegration analysis are given in Chapter Five. Our results are given in Chapter Six. Chapter Seven outlines the main findings of our study.

CHAPTER 2. MODELS OF PRICE LEVEL DETERMINATION

This chapter derives a pure exchange economy with money in the utility function and studies price level determination under Ricardian and non –Ricardian monetary and fiscal policies for three different cases.

The Model

There are infinitely lived agents with an endowment of y_t units of consumption good in each period. Government purchases g_t, yield no utility to the consumers. In each period t the consumer chooses the level of consumption. c_t, the nominal holdings of fiat currency. M_t, and the nominal holdings of government bonds, B_t. Fiat currency does not earn interest. However, consumers earn a gross nominal rate of interest, R_t for their holdings of nominal government bonds. Real balances, m_t, are defined as the ratio of nominal balances, M_t, and the price level, p_t. Similarly, the real value for government bonds, b_t, is defined as the ratio of nominal bond holdings, B_t, and the price level, p_t. At each period t, the consumer pays tx_t units of consumption good in lump sum taxes. Consumers have separable utility functions for money and consumer goods, and the utility is discounted at the constant rate of $\beta \in (0,1)$.

Given the initial wealth and the sequences of price, interest rate and net taxes, the representative consumer chooses the sequence of the end of period bond holdings $\{B_t\}$, the end of period money holdings, $\{M_t\}$, and the level of consumption $\{c_t\}$ to maximize his expected lifetime utility (1) with respect to the sequence of budget constraint (2), the transversality condition (3) and the nonnegativity constraints:

$$\text{Max } E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, \frac{M_t}{p_t}\right) \tag{1}$$

s.t

$$p_t c_t + p_t x_t + B_t + M_t \leq p_t y_t + R_{t-1} B_{t-1} + M_{t-1} \qquad t = 0, 1, 2, \ldots \tag{2}$$

$$\lim_{T \to \infty}\left(\prod_{\tau=0}^{T-1} R_\tau^{-1}\right) W_T = 0 \tag{3}$$

$$M_t \geq 0, \, c_t \geq 0$$

where the period t+1 nominal wealth, W_{t+1}, is defined by

$$W_{t+1} = M_t + R_t B_t \tag{4}$$

Note that the utility function is twice differentiable, strictly increasing, strictly concave and B_t can be either positive or negative.[1]

In the initial period 0, the government has initial outstanding liabilities M_{-1} and B_{-1}. Therefore the period 0 nominal wealth is: $W_0 = M_{-1} + R_{-1} B_{-1}$

The first order conditions are:

- $\frac{1}{R_t} U_c(c_t, m_t) = \beta E_t\left[\frac{p_t}{p_{t+}} U_c(c_{t+1}, m_{t+1})\right]$, for t=0,1,2,....

Note that under the assumption of separable utility and unitary consumption growth, $c_t = c_{t+1}$, this equality implies the Fisher Equation: $R_t = \frac{1}{\beta} E_t\left[\frac{p_{t+1}}{p_t}\right]$

- $\frac{U_m(c_{t+1}, m_{t+1})}{U_c(c_{t+1}, m_{t+1})} = R_t - 1$, for t = 0, 1, 2,

- $p_t c_t + p_t x_t + B_t + M_t \leq p_t y_t + R_{t-1} B_{t-1} + M_{t-1}$, for t = 0, 1, 2,...

where, $m_t = \frac{M_t}{p_t}$ and $b_t = \frac{B_t}{p_t}$.

---

[1] $B_t > 0$ implies that the government is borrowing from the household and $B_t < 0$ implies that the governmet is loaning to the household.

In the case of a log linear utility function:

$$U\left(c_t, \frac{M_t}{p_t}\right) = \log c_t + \log\left(\frac{M_t}{p_t}\right)$$

the first order conditions are, for $t = 0, 1, 2, \ldots$

$$\frac{1}{R_t}\frac{c_{t+1}}{c_t} = \beta E_t\left[\frac{p_t}{p_{t+1}}\right] \tag{5}$$

$$\frac{M_t}{p_t} = c_t\left(\frac{R_t}{R_t - 1}\right) \tag{6}$$

and

$$c_t = y_t - tx_t - \frac{B_t}{p_t} - \frac{M_t}{p_t} + R_{t-1}\left(\frac{B_{t-1}}{p_t}\right) + \left(\frac{M_{t-1}}{p_t}\right) \tag{7}$$

Government expenditure is financed by direct lump sum taxes and by money and bond seignorage. Hence, the sequence of government budget constraints is defined as:

$$p_t g_t - p_t tx_t = M_t - M_{t-1} + B_t - R_{t-1} B_{t-1} \quad \text{for } t = 0, 1, 2, \ldots \tag{8}$$

There is no capital accumulation. In this pure exchange economy, the sequence of aggregate resource constraint is defined as, the total endowment in each period being consumed by the fiscal sector and the private agents:

$$y_t = c_t + g_t \quad \text{for } t = 0, 1, 2, \ldots \tag{9}$$

In equilibrium the resource constraint (9) and the budget constraints (7) and (8) have to be satisfied. Note that any two of these constraints imply the third constraint. Therefore, it is possible to work with any two of equations (7), (8) and (9).

The optimisation problem includes the transversality condition.[2] The sequence of single period budget constraint, the sequence of aggregate resource constraint and the transversality condition lead to the intertemporal budget constraint[3]

$$\frac{W_t}{p_t} = \sum_{j=0}^{\infty}\left(\prod_{s=t}^{t+j-1} r_s^{-1}\right)(tx_{t+j} - g_{t+j} + \Omega_{t+j} \, m_{t+j})$$ (10)

where,

$$\Omega_{t+j} = \frac{R_{t+j} - 1}{R_{t+j}} \quad \text{and} \quad r_s = \frac{p_s}{p_{s+1}} \, R_s.$$

The intertemporal budget condition (10) implies the equality between the real value of household sector wealth and the present value of the expected future government surpluses.[4] The importance of this equation for equilibrium depends on the fiscal/ monetary policy.

The equilibrium is defined as a sequence of $\{c_t\}$, $\{R_t\}$, $\{p_t\}$, $\{M_t\}$, $\{B_t\}$, $\{g_t\}$, $\{tx_t\}$ and $\{W_{t+1}\}$ that are consistent with the monetary –fiscal policy and satisfy equations (5),

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[2] The transversality condition requires the household to fully utilize lifetime wealth. A violation of this condition implies that there is still room for a household to increase its lifetime utility.

[3] The intertemporal budget constraint is obtained from the single budget constraint by summing it up for each time period over the infinite horizon

[4] The same equation can be derived from the government budget constraint. Following Leeper (1991) the relation between equations (7), (8) and (9) allows us to represent equation (10) in terms of government debt and surplus. Using the transversality condition –which recognizes the fact that the fiscal authority can sell as much bond as he wants as long as the present value of debt is equal to zero– and equation (8) the government intertemporal budget constraint is defined as:

$$\frac{B_t}{p_t} = \sum_{j=0}^{\infty}\left(\prod_{s=t}^{t+j-1} r_s^{-1}\right)(tx_{t+j+1} - g_{t+j+1} + \frac{M_{t+j+1} - M_{t+j}}{p_{t+j+1}})$$ (10')

The real value of government debt is equal to the present discounted value of future government revenue –which is the tax revenues net of expenditures and seignorage -

Defining S_t as the present value of government surplus from money seignorage and tax revenue net of expenditures, equation (10') can be written as:

$$\frac{B_t}{p_t} = S_t$$ (10'')

(6), (8), (9), (10) together with the initial conditions and the exogenous output sequence $\{y_t\}$ [5].

The fiscal –monetary policy is defined as sequences $\{g_t\}$, $\{R_t\}$, $\{B_t\}$, $\{M_t\}$ and $\{tx_t\}$ such that the government flow budget constraint (equation (8)) is satisfied in every period. The policy can be Ricardian or non –Ricardian according to the relation between the real value of the government debt and the price level.[6] The policy is defined as Ricardian, if it ensures that the government budget constraint is satisfied for all price sequences. In case of a Ricardian policy the monetary and fiscal policy variables are determined endogenously by prices guaranteeing the solvency of the intertemporal government budget constraint. Therefore, the intertemporal budget constraint is satisfied for all price levels. However, for non –Ricardian policy the intertemporal budget constraint is an equilibrium condition, not satisfied for every price sequence. For non – Ricardian policies the policy authority is not constrained by the solvency of the intertemporal government budget constraint. Solvency will, however, emerge as part of the economy's equilibrium solution.

Defining the intertemporal government budget constraint as $\dfrac{B_t}{P_t} = S_t$, it is seen that a non-Ricardian fiscal policy argues that the out of equilibrium real values of the surplus are not equal to the real values of debt.[7] This does not mean that the government does not care about the budget constraint. It is simply that for non-Ricardian polices the level of surplus is set before the price level is determined. Any threat to the solvency of the budget constraint is confronted by the market mechanism moving the price level.

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[5] Under perfect foresight equilibrium, which we will assume for the empirical tests, the equilibrium sequences are such that the money and bond supplied by the government equal to the money and bond demanded by the household.

[6] Woodford (1994)

[7] A Ricardian fiscal policy argue that the fiscal policy sets the sequence of $\{B_t\}$ or $\{tx_t\}$ such that the real value of surplus defined as money seignorage and primary surplus is equal to B /p for all price levels.

"Under a non –Ricardian rule, the government moves before the "Walrasian auctioneer" does, so that the auctioneer is forced to call prices that are consistent with the real surpluses announced by the government." (Bassetto, 2001)

Under a Ricardian rule, it is important to indicate whether the monetary or fiscal authority is dominant, in the sense that the authority sets its policy variables exogenously. Leeper (1991) defines a policy as active if the authority sets its policy variable without constraining itself by the actions of the other authority. Similarly, a policy is referred to as passive if the policy authority is required to set its policy variable in response to the actions of the other authority to satisfy the intertemporal budget constraints in the system.[8]

"Because an active authority is not constrained by current budgetary conditions, it is free to chose a decision rule that depends on past, current, or expected future variables. A passive authority is constrained by consumer optimisation and the active authority's actions, so it must generate sufficient tax revenues to balance the budget."
(Leeper, 1991)

Monetarist Theory

The monetarist theory of price level determination is based on the argument that the monetary authority has total control over prices. Under a Ricardian monetary –fiscal policy, where the monetary authority is dominant, the fiscal authority determines the sequences of $\{g_t\}$, $\{tx_t\}$ and $\{B_t\}$ such that the government budget constraint is satisfied at all price levels. Therefore, with a monetary policy controlling the sequences of money

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[8] When a monetary/fiscal authority follows an active policy it is regarded that the monetary/fiscal authority is dominant.

supply {M$_t$}, the price level path is determined from the money market equilibrium condition, independent of the fiscal policy variables.

The monetarist theory on price level determination is defined by active monetary and passive fiscal policy operating within a Ricardian framework. Thus, following the basic model developed previously, under a constant level of consumption, equations (5) and (6) and the monetary policy setting the money supply determines the price level sequence independent of the fiscal policy variables:

$$p_t = \left(\frac{\beta M_{t-1} p_{t-1}}{M_{t-1} - c p_{t-1}} \right) \tag{11}$$

As is seen from equation (11) additional constraints are needed to uniquely determine the price level.

Unpleasant Monetarist Arithmetic

Sargent & Wallace (1981) showed that it is possible for the fiscal authority to affect the level of prices even with Ricardian policies. The "Unpleasant Monetarist Arithmetic" of Sargent and Wallace (1981) is a result of a Ricardian policy with active fiscal and passive monetary policies. They argue that if the fiscal authority acts in a dominant fashion and, for example, sets a constant level of government expenditures net of taxes (g −tx) and keeps the real value of government bonds (B/p) at a predetermined level, a Ricardian environment requires the monetary authority to be a follower.[9] That is, the monetary policy responds to a dominant fiscal policy by setting a growth rate of money to

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[9] A policy authority following an active policy is regarded as dominant and the policy authority forced to follow a passive policy is regarded as a follower.

generate the money seignorage necessary to satisfy the government budget constraint.[10]

Hence, contrary to the standard monetarist argument, an "...... expansionary fiscal policy

is inflationary" (Sargent and Wallace, 1981). Carlstrom and Fuerst (1999) define this

model of fiscal dominancy as a weak form of the fiscal theory of the price level (FTPL)

due to the fact that although the price level is still driven by the monetary authority, the

growth rate of money is now a function of the fiscal policy variables, g, tx and B.[11]

In their paper, Sargent and Wallace define the monetary policy as money supply

targeting rules. However, for our purposes it will be more convenient to define it as an

interest rate targeting policy that is formulated to satisfy the government budget

constraint. Hence, the interest rate rule for the monetary authority is defined by the

government budget constraint given the household demand conditions and the fiscal

policy.

Recall that for a constant level of primary surpluses, $D = (g - tx_t)$ and a constant real

government debt b, the government flow budget constraint is:[12]

$$D = m_t - m_{t-1}\left(\frac{p_{t-1}}{p_t}\right) + b - R_{t-1}\left(\frac{p_{t-1}}{p_t}\right)b \qquad (8')$$

Given fiscal policy variables and the demand conditions, (5) and (6), equation (8)

solves for the path of the interest rate as a function of the fiscal policy variables:[13]

$$R_t = 1 + \frac{R_{t-1} - 1}{(R_{t-1} - 1)\left(\frac{D}{c} - \frac{b}{c}\left(1 - \frac{1}{\beta}\right) - 1\right) + \frac{1}{\beta}} \qquad (12)$$

---

[10] Sargent and Wallace define monetary policy as money growth rate policies, $M_t = \theta M_{t-1}$. The analysis is based on constant government expenditures, constant rate of interest being greater than the growth rate of population and constant nominal bonds, which is determined historically.

[11] Dominant fiscal policies argue for central bank independence since in that model "The central bank is driven by the fiscal authority" (Carlstrom and Fuerst, 1999)

[12] Under the aggregate resource constraint: $y_t = c_t + g_t$ the household and government budget constraints are the same.

[13] A detailed analysis is given in appendix A

However, stability analysis and the comparative statics show that the fiscal policy does not affect the (stable) steady state equilibrium interest rate.

Note that the rate of inflation, $\pi_t = \dfrac{p_t}{p_{t-1}}$ is now a function of the fiscal policy variables, D and b. as well as the monetary policy variable, R:

$$\pi_t = \beta + \frac{\beta(R_{t-2} - 1)}{(R_{t-2} - 1)\left(\dfrac{D}{c} - \dfrac{b}{c}\left(1 - \dfrac{1}{\beta}\right) - 1\right) + \dfrac{1}{\beta}} \tag{13}$$

"One of the most important implications of this theory is the possibility that tight money today could increase *today's* price level! That is. a low money supply today necessitates increased inflation tomorrow. implying – if money demand is sufficiently elastic – a high price level today.......Low money today directly lowers current prices. But there is an additional. indirect effect – the higher future inflation necessary for budget balance increases the nominal interest rate, lowering real money demand today. The latter effect drives up today's prices and overwhelms the former if money demand is sufficiently interest elastic." (Carlstrom and Fuerst, 1999)[14]

Fiscal Theory of Price Level

Ricardian policies do not permit a dual dominancy of monetary and fiscal authorities. However, the fiscal theory of price level argues that in a non –Ricardian policy environment it is possible for both the fiscal and the monetary authorities to follow an active policy.

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[14] For that to hold the household money demand must be a function of future price levels as well as today's level of prices.

The fiscal theory of price level is a rather new approach to monetary economics developed by Woodford (1995), Leeper (1991) and Sims (1993) to answer the following questions:

- How is the price level determined in the case of endogenous money, which occurs with free banking and interest rate pegging?

- Is it true that monetization is not a significant financial tool for governments who depend less on money seignorage?

The weak relation between monetary aggregates and the inflation rate in the U.S. and the fact that the U.S. inflation rate is stable even though the U.S. follows an endogenous money policy are the two facts that Woodford (1995) tries to explain.[15] The model is formed to capture the impact of fiscal policy on the price level, which is believed to be the missing point of the conventional monetarist view. Woodford does not argue that the equilibrium conditions of the quantity theory of price level are irrelevant, but rather incomplete.[16] Contrary to the monetarist view that inflation is being driven only by monetary aggregates, he argues that the price level is determined from the government budget constraint as the ratio of the nominal value of debt to the present value of expected future surpluses.

Woodford argues that the LM equation defines the equilibrium interest rate differential (in case money is exogenous) or the money supply (in case money is endogenous) rather than the equilibrium price level as monetarist theory suggests.[17]

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[15] Cochrane (1998) presents the US the growth rate of base money, M1 and M2 together with the consumer price index. He argues, that the "... variation in inflation has essentially nothing to do with the history of monetary aggregates. The swings of inflation in the 1970's and especially the dramatic end of inflation in the 1980's occurred without any obvious corresponding changes in monetary growth."

[16] Under the quantity theory of money there are an infinite number of price path solving for the equilibrium conditions.

[17] "With an interest elastic demand and fixed supply, money demand can still determine the expected rate of inflation or expected price level, but it does not determine the (ex-post) price level. The government budget constraint then determines the price level" Cochrane (1998)

Hence, with monetary policy following a pegged interest rate policy and fiscal policy

being non-Ricardian, the price level will be determined by the present value of

government liabilities.[18] This condition implies that "the current money supply and its

expected future path are irrelevant for the determination of the equilibrium price level."

Woodford (1995) A monetary shock will effect the price level eventually ".....only as a

result of the eventual effects of monetary policy upon the size of the total government

liabilities, which then affects the price level through the fiscal policy rule. And even in

this case. it is arguable that such effects upon the price level as occur are due to fiscal

effects of policy change, rather than upon the mere fact that households are forced to hold

a different quantity of money; for the price level grows in proportion to the growth of

total government liabilities, and not in proportion to growth of the monetary component

of those liabilities." (Woodford, 1995)

Given the private sector's problem, the monetary policy as an interest rate peg, and

fiscal policy as an exogenous budget debt/surplus value, equation (5) drives the interest

rate rule:

$$R_t = \frac{1}{\beta} \frac{p_{t+1}}{p_t}$$
(5')

And, the money supply is driven by equation (6'):

$$\frac{M_t}{p_t} = c\left(\frac{p_{t+1}}{p_{t+1} - \beta p_t}\right)$$
(6')

Hence, the intertemporal government budget constraint together with the demand

conditions and the monetary policy solves for the price level:[19]

---

[18] Woodford, notes that the impact of fiscal policy on the equilibrium value of money is consistent with the findings of Sargent since the value of money, which is a part of government debt, depends upon the expectations of the households on the debt flow to back it.

[19] A detailed analysis is given in appendix A

$$p_t = p_{t-1} \left(\frac{\sum_{j=0}^{\infty} \beta^{j+1}(c-D)-b}{\sum_{j=0}^{\infty} \beta^{j}(c-D)-\dfrac{b}{\beta}-c} \right)$$

(14)

Contrary to the monetarist view that inflation is being driven only by monetary aggregates, in a non –Ricardian environment with active fiscal and monetary policies, the price level is only a function of fiscal policy variables. It is determined from the government budget constraint as the ratio of the nominal value of debt to the present value of expected future surpluses[20][21][22].

Ricardian policies assume that the Ricardian Equivalence Theorem holds. That is, the fiscal policy does not create any wealth effects.[23] However, this is not the case for non – Ricardian policies. In the case of active monetary and fiscal policies, the fiscal policy does create a wealth effect since an increase in the value of government bonds affects the households' lifetime budget set.

"The way the fiscal disturbances affect the price level is through a wealth effect upon private consumption demand. A tax cut not balanced by any expectation of future tax increases would make households perceive themselves to be able to afford more lifetime consumption, if neither prices nor interest rates were to change from what would have been their equilibrium values in the absence of the tax cut. This would lead them to demand more goods than they choose to supply (both immediately and in the future). The resulting imbalance between the demand and supply of goods drives up the price of goods

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[20] The price level is determined from the intertemporal government budget constraint (equation (10)):

$$p_t = W_t \left(\sum_{j=0}^{\infty} \left(\prod_{s=t}^{t+j-1} r_s^{-1} \right) (tx_{t+j} - g_{t+j} + \Omega_{t+j} m_{t+j}) \right)^{-1}$$

[21] Recall that the monetarist theory solved for a path of the price level. Additional constraints have to be imposed on the model for equation (13) to solve for today's price level. However, the FTPL solves for today's price level without imposing additional restrictions on the path of prices.

[22] The comparative statistics shows that a positive fiscal shock has a positive impact on the price level

[23] "Government bonds are not net wealth" (Barro, 1984).

until the resulting reduction in the real value of households' financial assets causes them to curtail demand (or increase supply) to the point at which equilibrium is restored." (Woodford, 1998)

In his studies Woodford takes these strong arguments on the impact of fiscal policy, one step further and argues that the fiscal theory of price level works under any type of monetary regime unless there is a Ricardian fiscal policy, which is a 'special' case.

Theoretical Studies on Fiscal Theory of Price Level

One significant force behind the FTPL is the ability and will of the fiscal authorities to use money financing. Hence, it is reasonable to ask if governments with less dependence on money financing could still choose to impose a higher price level. The studies of Leeper (1991) and Bergin (2000) have important results on that issue.

Leeper (1991) argues that the average level of money seignorage is not a factor on deciding the financing method of the debt. Hence, it may be misleading to argue that economies with a low dependence of money seignorage do not choose money financing. He specifies cases where money financing is an option for governments even when all their debt is backed by taxes. For this study he uses an FTPL framework with monetary and fiscal policy, given as:

- Monetary authority sets the interest rate, R_t as a function of inflation rate:

  $R_t = a_0 + a \pi_t + \varepsilon_t$

- Real tax revenue, tax_t as a function of government debt, b_{t-1}

  $tax_t = \alpha_0 + \alpha \, b_{t-1} + \eta_t$

where, ε_t and η_t are AR(1) processes.

These policy functions along with the solution to the private sector's optimization problem define a system of inflation, π_t and real debt, b_t:

$$E_t \, \tilde{\pi}_{t+1} = \alpha \, \beta \tilde{\pi}_t + \beta \varepsilon_t$$

$$\varphi_1 \, \tilde{\pi}_t + \tilde{b}_t + \varphi_2 \, \tilde{\pi}_{t-1} - (\beta^{-1} - \gamma) \tilde{b}_{t-1} + \varphi_3 \varepsilon_t + \psi_t + \varphi_4 \varepsilon_{t-1} = 0$$

where, $\tilde{\pi}_t$ and $\tilde{b}_t$ are fluctuations from the corresponding steady state values and,

$$\varphi_1 = \frac{c}{R-1}\left[\frac{1}{\beta\pi} - \frac{a}{R-1}\right] + \frac{b}{\beta\pi}$$

$$\varphi_2 = \frac{a}{\pi}\left[\frac{c}{(R-1)^2} - b\right], \qquad \varphi_3 = -\frac{c}{(R-1)^2}, \qquad \varphi_4 = \frac{\varphi_2}{a}$$

c, R, π and b are the steady state values of consumption, nominal interest rate, nominal rate of inflation and the real debt.

This system can be used to discuss different scenarios of price level determination, one of which shows that, with active fiscal and passive monetary policy –i.e., under a pegged interest rate and constant tax revenue- the price level is determined by the government budget constraint.[24] "Under pegged nominal interest rates and active fiscal behaviour, monetary policy's effect on prices depends on how the fiscal authority adjusts direct taxes in response to real debt movements. When taxes are unresponsive to debt, unanticipated monetary contractions immediately raise nominal interest rates and real debt and lower real balances. Prices respond with a lag. If future direct taxes rise (fall) with increases in real debt, the contraction lowers (raises) current inflation." (Leeper, 1991)

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[24] "..an active authority is not constrained by current budgetary conditions, it is free to choose a decision rule that depends on past, current or expected future variables. A passive authority is constrained by consumer optimisation and active authority 's actions, so it must generate sufficient tax revenues to balance the budget" Leeper (1991)

Bergin (2000) follows the same approach to address the importance of money financing in the case of a monetary union. He formulates a two-country model with a common central bank. In this model, the central bank controls the money supply through open market operations. The national governments receive transfers from the central bank and adjust lump-sum taxes to finance their deficits.[25] The rational expectations equilibrium conditions of the model lead to important arguments for a monetary union. First of all, under certain risk sharing assumptions, it is not required for all the member countries to have fiscal solvency for price stability. It is argued that under perfect insurance the debt of a country is absorbed by the surplus of another country. It is important to point out the fact that this proposition leads to important wealth effects. Bergin also argues that one bad apple could spoil the whole bundle. Even if the common central bank refuses to issue new money, the governments with large debt would still follow irresponsible polices and campaign for higher price levels. This is a problem for other countries too since an increase in the unbacked debt of a government not only increases the price level in that country but it increases price levels all through the monetary union.

In contrast to these arguments, the study by Dupor (2000) on open market models indicates that the FTPL is not sufficient to pin down the equilibrium price level. He studies the determination of the exchange rate in a two-country set up with dominant fiscal policy and nominal interest rate pegging. His model allows for households to exchange goods, money and government bonds under a no –arbitrage condition in the bonds market. His solutions for the cases of substitutable and non-substitutable currencies indicate that, " the nominal exchange rate is indeterminate if both governments peg the

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[25] It is assumed that the transfers to the national governments are symmetric

interest rate on domestic bonds." (Dupor, 2000) This conclusion for the indeterminacy of the price levels of each country and the exchange rate is in contrast to the arguments of the FTPL.

To see if the FTPL is an acceptable model, Carlstrom and Fuerst (1999) analyzes the reliability of the assumptions of two different versions of the FTPL, which they name 'strong form' and 'weak form' FTPL. Both forms of the FTPL assume a dominant fiscal policy for price level determination. However, they differ in the way the monetary policy is applied. "Weak form FT posits that inflation is indeed a monetary phenomenon, but that money growth is dictated by the fiscal authority. Strong form FT, on the other hand, argues that even if money growth is unchanged, fiscal policy independently affects the price level and the inflation rate." (Carlstrom and Fuerst, 1999). Working with separable preferences and an interest rate pegging policy, the partial and general equilibrium analyses of the weak and strong form FT indicate that, strong form FT requires unrealistically large interest elasticity of money demand and output elasticity of money demand for fiscal policy to be able to determine the price path.

The dependability of the FTPL for real world analyses is also questioned in the study of Kocherlakota and Phelan (1999). They argue that that it is more reasonable to believe in the monetarist theory of price determination since it is not 'logical' for governments to choose an inflationary outcome.[26]

McCallum (2001) is against FTPL due to the definition of the fiscal policy that is used.[27] He argues that the fiscal policy variable is actually the bond supply to the public, not the primary deficit or surplus. He shows that there is a monetarist type solution for the

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[26] Their arguments on FTPL are based on certain examples for monetary and fiscal policy.

[27] His results are independent of the elasticity of money demand and any specific form of price path that has been required in previous studies.

price level in an FTPL framework, where the bond supply is taken as the fiscal policy variable.

Cushing (1999) is another attack on the FTPL. He modifies the original model used by the advocates of FTPL and assumes that the households face a certain probability of death.[28] Households do not leave bequests but they receive payments from insurance companies. The insurance companies finance these premium payments by collecting the financial assets of the deceased. The equilibrium conditions of this model create a system of money supply, bond supply and the price level, which is a function of the interest rate, government spending, probability of death, money supply and bond supply. Cushing solved the model for the cases of Ricardian and non –Ricardian consumers and concluded that the price level is indeterminate.[29]

In their study on the FTPL, Schmitt-Grohe and Uribe (2000) are rather careful on the conditions where this theory might work. They model a cash-in-advance economy, to study the dependability of the balanced budget requirement as an anchor for price stability. Under the assumption that the government is not allowed to finance the deficit with money seignorage, the price level is determinate in case the primary rather than the secondary budget surplus/deficit is taken to be exogenous.

To conclude, the FTPL is a controversial theory.

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[28] The probability of death is used to differentiate between Ricardian and non-Ricardian policies as well as to create a divergence between the asset holdings of agents. For the Ricardian consumer the probability of death is zero

[29] He argues that the FTPL is based on the two unrealistic assumptions; i) the government debt converges and ii) the future inflation is constant.

Empirical Studies on Fiscal Theory of Price Level

These theoretical models of the price level determination were tested by many

empirical studies. Some of these studies followed a time series approach and others

followed a structural approach.

Structural studies use the government budget constraint to model the inflation rate as

a function of government debt. Metin (1998), Ruge-Marcia (1999) and Cardosa (1992)

are some recent examples of this type of empirical study.

Metin (1998) estimated the following equation for the inflation rate in Turkey as a

function of the Turkish budget deficit and output growth:[30][31]

$$\Delta p = a + \delta B - \Delta y$$

where, Δy is the output growth rate. B is the scaled budget deficit; $B = \dfrac{G-T}{H}$, where G is

public sector expenditures, T is revenues, and H is base money.

The regression results and tests for cointegration indicated that government deficits

have a significant positive impact on the inflation rate.[32]

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[30] The price level equation is a result of the modification of the government budget constraint;

$$\frac{G-T}{PY} = \frac{\Delta H}{PY} \tag{M1}$$

where, Δ is the difference operator,G is public sector expenditures, T is revenues, H is base money, P is prices and Y is real income.

In steady state,

$$\Delta H^{*} = \frac{\Delta H}{PY} - H^{*}(\Delta p + \Delta y) \tag{M2}$$

where, p and y are log(P) and log(Y) respectively.

Solving equation (M2) for $\dfrac{\Delta H}{PY}$ and using it in equation (M1) gives the budget constraint;

$$\frac{G-T}{PY} = \Delta H^{*} + H^{*}(\Delta p + \Delta y) \tag{M3}$$

Then, equation (M3) is solved for price level.

[31] Annual data over the sample period 1950-1987.

[32] "......an increase in the scaled budget deficit immediately increases inflation." Metin (1998)

Ruge-Murcia (1999) used annual data for Brazil over the sample period 1940 –1988,

finding that the level of expenditure is an important factor for the level and volatility of

the inflation rate.[33] [34]

Although these studies are for countries with huge government debt, there are

studies for the U.S. indicating that non monetary factors have a significant impact on the

price level.

Ahking and Miller (1985) used a multi- equation time series approach to study the

relation between inflation and the public debt. In this study, they followed a three-stage

procedure where base money growth, government deficits and inflation were treated as

endogenous variables. The three–stage OLS analyses for the U.S. economy showed that

government deficits have an important effect on the inflation rate.[35] [36]

Dhakal, Kandil, Sharma and Trescott (1984) focused on demand-pull/ cost-push

theories for the causes of inflation.[37] First, they estimated a vector autoregression (VAR)

model of the money stock M1, producer price index, interest rate and the gross national

product to test the validity of the monetarist approach to explaining the inflation rate.

Second, they created three new models by adding the government debt, wage rate and

energy prices sequentially to the original model to search for the non-monetarist

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[33] In that study Ruge- Marcia developed a dynamic model for inflation in case of monetization of the government debt, which is defined as the difference between government expenditures and the tax revenue. Tax revenue is modelled as a backward looking process such that it is influenced by the past inflation rates. He assumed a Cagan model for money demand and that the money supply is an endogenous process indexed to the government debt. The government debt is endogenous and assumed to be partially financed by an increase in money supply. The government expenditure is regarded as the fiscal policy variable and is assumed to follow a stationary second order autoregression process. Following a rational expectations approach the model is solved for the inflation rate, which is a function of past inflation rates and the discounted values of current and expected future government expenditures.

[34] It is assumed that the money supply and budget deficit are endogenous but government expenditure is an exogenous variable.

[35] Annual data over the sample period of 1950-1980

[36] "...the deficits cause inflation in the 1950's and the 1970's but not in the 1960's. The quantitative effect of deficits on inflation is small in the 1970's but not in the 1950's" (Ahking and Miller, 1985)

[37] Quarterly data over the period of 1957-1991

determinants of inflation.[38] For each of these four models they used the results of Granger

causality tests and the specific gravity criterion of Caines, Keng and Sethi (1987) to solve

the identification problem. However, the results could not resolve the conflict between the

monetarist view and recent developments on inflation rate determination. The monetarist

model indicates that the money supply has a strong effect on the price level. However, the

VAR results also indicate that government debt, the wage rate and energy prices Granger

cause the inflation rate. The VAR analysis indicates that the public debt has a significant

effect on the inflation rate through aggregate demand, even when monetization was not an

issue.

The study by Cardoso (1992) on the economies of Brazil and Mexico indicates that

fiscal policy is important for the stability of inflation in open economies. The empirical

analysis of an open economy model showed that fiscal consolidation is very important for

the success of disinflationary programs for economies with large fiscal debt.[39] Moreover,

it indicated that the existence of a huge external debt is a leading factor of the inflationary

impact of government debt.

The importance of the definition of the government debt for the effectiveness of

fiscal policy is the central point in the study of Abizadeh and Yousefi (1998). They

argued that the inconclusive results of empirical studies on the impact of the budget

deficit on the inflation rate is due to the limited definition of public debt for closed

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[38] An increase in energy prices and wage rate are the cost-push factors of inflation. Such a policy increases the cost of production, leading to a lower level of output. The consequent decline in output will result with an increase in the price level. However, any change in the budget deficit is a demand-pull factor of inflation. An increase in budget deficit implies higher government spending and lower taxes creating a positive impact on demand.

[39] The definitions for inflation rate, interest rate, balance of payments and the domestic credit creation together with the assumption that the public debt is financed by domestic credit (without bond financing) and external borrowing (no private external borrowing), the inflation rate is defined as a function of money supply, the share of the primary budget deficit in output and the share of net exports in output.

economy models.[40] They analysed an open economy IS –LM model by VAR. The need to solve for multicollinearity between monetary and non monetary variables of inflation, led to a linear model of the domestic inflation rate (P_t) as a function of lagged real gross domestic product (Y_{t-1}), the real deficit (DF_t), the foreign rate of inflation (q_t) and the domestic money supply ($M1_t$):[41]

$$P_t = f [Y_{t-1}, DF_t, q_t, M1_t]$$

The OLS and ML estimation results showed that "…budget deficits have no significant bearing on the rate of inflation".[42]

The study by Canzoneri, Cumby and Diba (1998) also supports the monetarist view of the US inflation rate. They focus on the two opposite approaches to price level determination, –money or fiscal dominant regimes.[43] The basic assumptions of these regimes are important for the effect of monetary / fiscal policies on the price level.[44] Hence, in order to choose the monetary policy for price stability it is necessary to decide on the right nominal anchor for the economy: fiscal dominance (FD) or monetary dominance (MD). VAR methods are used to distinguish between MD and FD regimes, and to see which regime the country has been following. They argue that the basic difference between the monetarist view and the FTPL lies in the way they look at the government budget constraint. In their closed economy model, the primary deficit is financed by bond or money seignorage. After scaling the variables of the customary government budget constraint by GDP, the model can be represented as:

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[40]In open economy models, the government spending is financed by the tax revenue, the money and bond seignorage and the net transfer payments to other governments.

[41]The principal components method used to solve the problem of multicollinearity

[42] Annual data for U.S. over the sample period 1951-1986

[43] Fiscal dominant regime is defined as the one in which the price level is determined independently of the monetary aggregates, by the solvency of the government budget constraint. In contrast, in the money dominant regime the price level is determined by the supply and demand of money.

[44] Applying a money supply rule would cause an over determined price level in case of FD regime. However, an interest rate targeting rule results with an undetermined price level in case of an MD regime

$$w_j = s_j + \alpha_j \, w_{j+1}$$

where, the ratio of liabilities to GDP, w_j, is equal to ratio of the surplus to the GDP, s_j, plus the discounted value, α_j, of the ratio of next period's liabilities. Under MD and FD there is a positive correlation between s_j and w_j. However, the relation between s_j and w_{j+1} distinguishes between FD and MD. Under the assumption that the debt is following a backward looking process, monetarist theory predicts a fall in w_{j+1} in case of an increase in surplus. In contrast, in the fiscal regime a positive innovation in s_j can have a zero, positive or negative effect on w_{j+1}, depending upon whether s_j has a zero positive or negative correlation with future surpluses and discount factors. For the empirical tests, the authors look at the impulse responses and the forecast variances for the effect of the ratio of current surplus/GDP on the ratio of next period liabilities/GDP. The results indicate that the surplus is not exogenous but is affected by the current level of liabilities and although the current surplus/GDP ratio is not negatively correlated with future surpluses, the liabilities/GDP has negative reaction to a positive shock in surplus/GDP. Hence, it is concluded that, "the post war US data strongly favours the MD regime over the FD regime".[45]

Cochrane (1998b) criticizes this conclusion and argues that the negative effect of higher surpluses on the real debt is not a monetary outcome but the result of a conscious decision by the fiscal authority to decrease the volatility of the inflation rate.

Cochrane (1998b) argues that the composition of the government debt is crucial for the effect of surpluses on the price level. Therefore, he extends the one period debt version of the fiscal theory of price level to include long-term debt. Hence, the government debt is a function of nominal bond prices and so it is also a function of

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[45] The VAR result indicated that the surplus is not exogenous but affected by the current level of the liabilities and although the current surplus/GDP ratio is not negatively correlated with future surpluses, the liabilities/GDP has negative reaction to a positive shock in surplus/GDP.

expected future price levels. His analysis shows that the fiscal authority is able to postpone inflation by choosing an appropriate debt structure. A fiscal authority following an optimal passive policy can smooth inflation with a long maturity debt structure rather than with a short maturity structure if the present value of the surplus is more volatile than the level of surplus.[46] Similarly, the fiscal authority following an optimal active policy will have the ability to exchange future inflation with a decrease in today's price level by devaluing long-term bonds unexpectedly.[47]

Cochrane argues that the fiscal authority follows an optimal policy to smooth the inflation rate. The fiscal authority has control over the maturity structure and the level of debt and has partial control over the surplus. The surplus structure, s_t is defined as the sum of the cyclical portion c_t -which the fiscal authority cannot control- and the controllable component, z_t. If z_t is a random walk process, $z_t = z_{t-1} + e_t$, then the government can choose e_t at each time. Hence, given the equation for the price level,[48]

$$(1-L)p_t = -[(1-\beta)(1-\beta\Phi\rho)/(1-\beta\rho)(1-\beta\Phi)] (1-L) c_t -e_t +(1-L)(L-\Phi\beta)B_t \quad (*)$$

the optimal fiscal policy is the one where the government chooses the maturity structure Φ, the level of nominal debt sequence $\{B_t\}$ and the sequence $\{e_t\}$ to minimize the inflation rate.

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[46]Cochrane describes the optimal passive policy as the one in which "...the government determines only the steady state level of debt and its maturity structure, and the government does not adjust debt in response to surplus shocks"

[47]Cochrane describes the optimal active policy as the one in which the government "....changes the amount and maturity structure in response to surplus shocks"

[48] Cochrane argues that the sequence of prices are a solution to the present value identity (except the case of no new debt) for a given sequences of debt and surplus. The present value identity is the equality of the real value of outstanding debt to the present value of net surpluses:

$$B_{t-1}/p_t + \sum_{j=1}^{\infty} \beta^j E_t (1/p_{t+j}) B_{t-1}(t+j) = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$$

Iterating this identity forward with geometric weights together with the equation of surplus leads to the price equation (*)

A comparison of the artificial time series generated by this optimal fiscal policy with the actual U.S. data shows some similarities. However, it is not a perfect match. Cochrane argues that the reason is that the optimal fiscal model is too successful in reducing the variation of the inflation rate.

Cochrane (1998a) studies the history of the U.S. inflation rate from an FTPL view, using a structural VAR model of prices, debt and the surplus which is modelled as a sum of a long run component (z_t) and a cyclical component (c_t).

The results of this analysis for the inflation rate are the same as those in the previous paper by Cochrane. The model fits nicely to the US data. Based on this finding Cochrane argues that, the government adjusts its budget to smooth the rate of inflation in case of a cyclical surplus shock. The government "...sells extra debt in recessions, raising revenue by so doing because it implicitly promises to raise subsequent surpluses" (Cochrane, 1998)

The basic difference between the monetary theory and the FTPL is the way that fiscal policy is modelled. Hence, distinguishing between the Ricardian and non –Ricardian policy is the first step to model a policy to stabilize the rate of inflation. Christiano and Fitzgerald (2000) propose two ways to choose between Ricardian and non –Ricardian policies:

"One is to try to extrapolate what is reasonable out of equilibrium behavior based on what we see in equilibrium. Another way is to view the FTPL as a starting point for natural set of auxiliary assumptions which do restrict time series data, and then test those assumptions." (Christiano and Fitzgerald, 2000)

The structural VAR models of Cochrane (1998) and Canzoneri, Cumba and Diba (1998) relied on this second approach to analyse the history of inflation. Their models

based on the arguments of FTPL, focused only on the relation between the public surplus

and debt.[49]

[49] The model Cochrane developed "captures only the part of inflation that are correlated with surplus and value" Cochrane (1998a)

CHAPTER 3. MULTI -EQUATION MODELS

Vector Autoregression Models

The main advantage of VAR analysis is that it can treat all variables symmetrically. All variables are treated as endogenous with the time path of each variable determined by the current and past values of themselves and the other variables. A general dynamic structural system can be written as a pth order vectorautoregression process;

$$D (L) X_t = \varepsilon_t \tag{16}$$

where, X_t is an n dimensional stationary vector stochastic process, $D (L)$ is a pth degree matrix polynomial in the lag operator L; $D_0 - D_1 L - D_2 L^2 - \ldots -D_p L^P$, with the D_i s being nxn matrices. D_0 is an nxn matrix of parameters on the contemporaneous endogenous variables and ε_t is an nx1 vector white noise process whose elements are contemporaneously uncorrelated structural disturbances.

Equation (16) is a structural/primitive vector autoregressive representation of the time series. Solving for X_t gives the vector autoregressive model in its standard reduced form:

$$A (L) X_t = e_t \tag{17}$$

where, $A(L) = I_n - A_1 L - A_2 L^2 - \ldots -A_P L^P = I_n - D_0^{-1}(D_1 L + \ldots + D_p L^p)$ and $e_t = D_0^{-1} \varepsilon_t$.

Note that the elements of e_t are serially uncorrelated error terms with zero mean and constant variance, but will typically be contemporaneously correlated since they are linear combinations of the structural disturbances, $\varepsilon_{1t}, \ldots, \varepsilon_{nt}$.

The variance covariance matrix for the error terms of equation (17) is given as:

$$\Sigma_e = E (e_t e_t')$$

Since the error terms in (17) are related to the structural shocks in (16) according to

$$e_t = D_0^{-1} \varepsilon_t ,$$

$$\Sigma_e = E (D_0^{-1} \varepsilon_t \varepsilon_t' D_0^{-1})$$

and so,

$$\Sigma_e = D_0^{-1} \Sigma_\varepsilon D_0^{-1}$$

where, Σ_ε is the diagonal variance covariance matrix of the structural disturbances.

Economic theory can be used to specify X_t and the appropriate lag length tests can be used to determine p, the order of the VAR.[50]

Due to the correlation between elements of X_t and ε_t, the structural VAR is not used for estimation purposes. This is not the case for the VAR in standard form. The ordinary least squares estimator of each equation in (17) will give consistent and asymptotically efficient estimates of the coefficients, variances and covariances. However, the structural VAR representation will be underidentified in the absence of additional constraints. This identification problem is solved by imposing at least $(n^2-n)/2$ restrictions on the structural model.

There are several different procedures commonly used to impose restrictions on the matrix, D_0.

- Sim's Methodology: This procedure uses the Choleski decomposition of Σ_e to impose a Wold recursive structure on the matrix D_0. The results can depend highly on the order of the variables. Dharmendra, Kandil, Sharma and Trescott (1994) used Sims Methodology to capture the effects of monetary and non-monetary factors on the inflation rate. Petrovic and Vujosevic (2000) applied a Wald recursive structure and a long run theoretical restriction on the

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[50] The likelihood ratio test, the AIC (Akaike Information Criterion) test and the SBC (Schwartz Bayesian Criterion) test can be used to determine the lag length of the autoregression process.

model of the price level and found that aggregate supply shocks are the causes behind the high inflation rate of Yugoslavia.

- Sims –Bernanke methodology: This procedure uses economic theory to impose restrictions on the matrix D_0. Domenech, Taguas and Varela (2000) use the Ricardian Equivalance Theorem to impose such restrictions on a bivariate model of national saving and the budget deficit. Kim and Roubini (2000) use theoretical restrictions on an open economy model to solve various puzzles (ie., liquidity puzzle, price puzzle, exchange rate puzzle and forward discount puzzle) created by previous studies.

- Blanchard –Quah decomposition: This decomposition uses restrictions on the permanent effects of certain shocks on the levels of I(1) variables. Hoffmaister and Roldos (2001) use long run restrictions to study the effects of domestic and external shocks on GDP for Korea and Brazil. In an unpublished paper Falk and Lee (2000) use Blanchard –Quah type restrictions to capture the effects of aggregate demand, aggregate supply and productivity shocks on the inflation and unemployment rates.

The standard VAR representation of the system is most helpful for estimation and forecasting purposes. However, for innovation accounting – impulse response analysis and variance decompositions- the structural shocks have to be used.

Since it is assumed that X_t is covariance stationary, this system can also be represented as a vector moving average process (VMA) in terms of the structural shocks, ε_t. This representation is especially useful to capture how structural shocks determine the dynamic behaviour of the system.

The VMA is obtained from the VAR by solving for X_t:

$A (L) X_t = e_t$

$X_t = [A (L)]^{-1} e_t$

Recall that $e_t = D_0^{-1} \varepsilon_t$. Then:

$X_t = [A (L)]^{-1} D_0^{-1} \varepsilon_t$

$X_t = C (L) \varepsilon_t$

where $C (L) = C_0 + C_1 L + C_2 L^2 + \ldots\ldots = \sum_{i=0}^{\infty} C_i L^i$. Each C_i is an n x n matrix satisfying the

conditions for stationarity and invertibility.

The effects of the innovations ε_t on the sequence of the variables of the system $\{X_t\}$

are determined by the elements of the C_i's.

The impulse response of variable m at time t+i to k^{th} shock at time t is:

$$C_{i, mk} = \frac{\partial x_{m,t+i}}{\partial \varepsilon_{t,k}} \qquad m, k = 1,\ldots\ldots,n \text{ and } i = 0,1,2\ldots\ldots$$

The plots of the impulse response functions track down the dynamic responses of the

variables in X_t to the various elements of ε_t.

Note that, the impulse response matrices C_0, C_1, ...are only identified from the VAR

parameters A_0, A_1, ...and, Σ_e. Hence, if the structural VAR is not identified, it is not

possible to recover the impulse response functions from the estimated standard VAR

model.

Another tool of innovation accounting is the variance decomposition table. The

elements of this table provide the relative importance of each type of structural shock in

explaining the dynamic behaviour of each element of X.

Recall the system in VMA form is:

$$X_t = (\sum_{i=0}^{\infty} C_i L^i) \varepsilon_t$$

$$X_t = \sum_{i=0}^{\infty} C_i \, \varepsilon_{t-i}$$

The h period ahead forecast error is given by:

$$X_{t+h} - E_t X_{t+h} = \sum_{i=0}^{h-1} C_i \, \varepsilon_{t+h-i}$$

where, $E_t X_{t+h}$ is the expected value of X_{t+h} given all the available information at time t. Thus, the mean squared error of the h period ahead forecast is:

$$MSE = E[(X_{t+h} - E_t X_{t+h})(X_{t+h} - E_t X_{t+h})'] = \sum_{i=0}^{h-1} C_i \Sigma_\varepsilon C_i'$$

Let $c_{i,ms}$ be the (m,s) element of the matrix C_i and let σ_s^2 be the variance of the disturbance s. Thus, the h step ahead forecast error variance of the mth variable is given as:

$$E(X_{t+h} - E_t X_{t+h})^2 = \sum_{i=0}^{h-1} \sum_{s=1}^{n} c_{i,ms}^2 \sigma_s^2 \qquad m=1,\ldots,n$$

The forecast error variance decomposition, Ψ shows the percentage of forecast error variance of mth variable explained by the kth shock, ε_{kt}.

Vector Error Correction Models

The previous analysis is based on X_t being a stationary, I(0) process. That is, all the variables in the system have a tendency to return to their long run mean level. If the X's are nonstationary in levels but stationary in first differences, then in general the previous analysis can be applied to the first differences of X_t. However, if the variables are nonstationary, and there exists a linear combination which is I(0), then these variables are cointegrated. In this case, the vector error correction model is more useful than the vector autoregression.

If a linear combination of I(1) variables is stationary, then these variables are cointegrated. In case of cointegration, the system should be defined as a vector error correction model.

Recall the reduced form is:

$$A (L) X_t = e_t \tag{17}$$

A (L) is a pth degree matrix polynomial in the lag operator L; A (L) = $\sum_{i=1}^{p} A_i L^i$ and e_t is white noise with zero mean and constant variance with a possibility that e_{it} is correlated with e_{jt}.

If X_t is an nx1 vector of I (1) variables and cointegrated of order (1,1) with r cointegrating vectors the Granger Representation Theorem says that:

(i)   rank of [A (1)] = r (< n)

and

(ii) $A(1) = \alpha\beta'$

where, α is an nxr matrix of speed of adjustment parameters and β is an n x r matrix whose columns represent the cointegration vectors.

Given the cointegration relations, we can rewrite equation (17) to get the Engle – Granger vector error correction representation:

$$\Delta X_t = \Gamma X_{t-1} + H (L) \Delta X_t + e_t \tag{18}$$

where, $\Delta = 1-L$ is the first difference operator, $H(L) = \sum_{i=1}^{p-1}\left(-\sum_{j=i+1}^{p} A_j\right) L^i$ and $\Gamma = -A$ (1).

Equation (18) states a linear combination of I(1) variables that is stationary. Rewriting equation (18):

$$\Gamma X_{t-1} = (I_n - H (L)) \Delta X_t - e_t \tag{19}$$

Since, all the variables on the right hand side of this equation are I(0), ΓX_{t-1} has to be

stationary too. ΓX_{t-1} is the error correction part of the system with $\Gamma = -A$ (1) being

composed of cointegration and the speed of adjustment vectors. $\Gamma X_{t-1} = -\alpha\beta' X_{t-1}$ corrects

the change in X_t due to last period's long run equilibrium error. Since each row of the

matrix Γ defines the relations between I(1) variables, $\Gamma_{11} x_{1t-1} + \Gamma_{12} x_{2t-1} + \ldots + \Gamma_{1n} x_{nt-1}$, the

rank of $\Gamma = r \in [0, n]$, determines the number of cointegration relations. If the rank of Γ is

zero, the model takes the reduced form VAR representation, $\Delta X_t = H$ (L)$\Delta X_t + e_t$ without

any cointegration relations between variables. However, if the rank of Γ is r, there are r

cointegration vectors and so there exist $k = n - r$ common trends.

Since the economic interpretations of VAR models are done through innovation

accounting tools, the data generating process has to be represented as a moving average

(MA) process. From equation (18):

$$\Delta X_t = Q \ (L) \ e_t \qquad (20)$$

where, $Q \ (L) = [(I_n - H(L)) - \Gamma \ L(1 - L)^{-1}]^{-1}$. Unfortunately, this representation is based

on the reduced form error terms, which are mutually dependent.

Let N be an nxn matrix such that $N\Sigma_e N'$ is diagonal. Given the relation between

structural and reduced form error terms, $\varepsilon_t = Ne_t$ the structural form can be represented

as:

$$\Delta X_t = S \ (L) \varepsilon_t . \qquad (21)$$

$\varepsilon_t = Ne_t$, $S \ (L) = Q \ (L) \ N^{-1}$, $S \ (1) = Q(1) \ N^{-1}$

where, $E(\varepsilon_t) = 0$ and $E(\varepsilon_t \varepsilon_t') = \Sigma_\varepsilon = I_n$.[51] Thus, $\Delta X_t = S(L)\varepsilon_t = Q(L)e_t$

---

[51] Engel Granger representation is based on the assumption that cointegration exists. Note that if
S(1) ≠ 0, {X_t} is nonstationary. Moreover, if there are n variables with r cointegration relationship (CI (1,1))
then it is true that: (i) rank of S(1) = n −r and (ii) $\beta'S(1) = 0$

Equation (21) will be used to analyse this system[52]. Thus, we need to obtain an estimate of matrices S(1) and Σ_ε. Note that, S(1) is the total impact matrix and S (1)ε_t produces the impulse responses.

To estimate the system and calculate the matrices Q(L) and so S(L), we used Warne (1993) version of Ganger representation theorem.

Warne (1993) generalizes Campbell and Shiller (1988)'s work of rewriting VEC model as a restricted VAR with n = 2 and r = 1.

Let $\alpha^* = \begin{bmatrix} 0 & \alpha \end{bmatrix}$ is an nxn matrix and M be an nxn non-singular matrix given by $\begin{bmatrix} R_k' & \beta' \end{bmatrix}$ where R_k is a kxn selection matrix such that $R_k = \beta_\perp'$ where $\beta'\beta_\perp = 0$, $\beta_\perp'\beta_\perp = I_k$ and $R_{i,k}S(1) \neq 0$ for all $i \in \{1,....,k\}$.

And the nxn matrix polynomials,

$$G(L) = \begin{bmatrix} I_k & 0 \\ 0 & (1-L)I_r \end{bmatrix}, \quad G_\perp(L) = \begin{bmatrix} (1-L)I_k & 0 \\ 0 & I_r \end{bmatrix}$$

Note that the estimated cointegration vector is sufficient to determine the matrices M and $G_\perp(L)$[53].

Premultiplying both sides of equation (18) with the matrix M,

M (I −H(L)) ΔX_t = M Γ X_{t-1} + M e_t

P(L) $y_t = z_t$ (17.1)

y_t = P(L)$^{-1}z_t$

where, P(L) = M [(I −H(L)M^{-1}G(L)+α^*L], y_t = $G_\perp$(L)MX$_t$, Me$_t$ = z$_t$, Σ_z = E(z$_t$z$_t'$)

---

[52] Engel Granger representation is based on the assumption that cointegration exists. Note that if S(1) $\neq$ 0, {X$_t$} is nonstationary. Moreover, if there are n variables with r cointegration relationship (CI (1,1)) then it is true that: (i) rank of S(1) = n −r and (ii) β'S(1) = 0

[53] A researcher can either estimate the cointegration vector, β' by maximum likelihood estimation technique or he /she imposes the steady state values of the relations between variables based on economic theory.

Note that, equation (17.1) is a VAR representation conditioned on the cointegration vectors. Warne (1993) calls this restricted VAR (RVAR).

Equation (17.1) provides a simple connection to VMA representation and thus, very useful for the purposes of estimation and innovation accounting. Note that, given the Granger representation theorem, $Q(1) = M^{-1}G(1)P(1)^{-1}M$. Therefore, the estimates of S (1) and Σ_ε can be obtained from the estimates of M, P(1) and Σ_z.

As in VAR models, the VECM has an identification problem too. To solve the identification problem King, Plosser, Stock, Watson, (1991) (KPSW) imposes two sets of restrictions on the system. The first set of restrictions distinguishes the set of permanent shocks from the temporary shocks and the second set of restrictions identifies each permanent shock –in case there is more than one permanent shock.

The first set of restrictions is imposed by the cointegration vector. When Γ has rank r, there exists $k = n - r$ common trends, which represent the permanent shocks. Therefore cointegration relations impose constraints on the matrix of long run multipliers, S (1) = $\sum_{i=0}^{\infty} S_i$ which make it possible to recognize the permanent shocks and so, to decompose the structural shocks as permanent and temporary: $\varepsilon_t = (\varepsilon_t^p, \varepsilon_t^t)'$. Here, ε_t^p is a kx1 vector of permanent shocks and ε_t^t is an (n–k)x1 vector of permanent shocks[54].

Although cointegration recognizes the set of permanent shocks it fails to distinguish among them. KPSW imposes the property of no –correlation between the permanent and temporary structural shocks (ε_t^p and ε_t^t) to isolate the dynamic response of X_t to each permanent shock.

---

[54] Recall that $S(1) = Q(1) N^{-1}$ and the nxn matrix N is chosen such that: i) the permanent and temporary innovations are independent and ii) the transitory innovations are mutually independent.

Assuming there are r cointegrated vectors and k common trends, ε_t^p is a k x 1 vector and:

$$S(1) = \begin{bmatrix} A & 0 \end{bmatrix}$$

where, A is an n x k matrix of long run multipliers of permanent shocks whereas, 0, an n x (n -k) matrix of zeros which are the long run multipliers for temporary shocks. To identify each independent permanent shock, KPSW impose the following structural form:

$$A = \begin{bmatrix} \tilde{A}\Pi \end{bmatrix}$$

where. $\tilde{A}$ is an nxk matrix with known parameters, which is chosen such that $\beta'\tilde{A} = 0$.

Therefore, the innovations to the trends have an economic interpretation[55]

Π is a k x k lower triangular matrix with ones on the diagonal. The relation, $AA' = Q(1)\Sigma Q(1)'$ and the composition of the matrix A, $A = \begin{bmatrix} \tilde{A}\Pi \end{bmatrix}$ solves for

$\Pi\Pi' = (\tilde{A}'\tilde{A})^{-1}\tilde{A}'Q(1)\Sigma Q(1)'\tilde{A}(\tilde{A}'\tilde{A})^{-1}$. However, in order to estimate we need to impose

additional restrictions on matrix Π such as Cholesky decomposition or Sims –Bernanke

decomposition. In this paper we used Cholesky decomposition of Π. It is crucial to note

that Cholesky decomposition of Π need not indicate a recursive structure on matrix A.

Matrix $\tilde{A}$ determines the effects of common trends in the system.

---

[55] Note that for $k =1, \Pi$ is an identity matrix, $A = \tilde{A}$.

CHAPTER 4. MODEL SPECIFICATIONS

Structural Vector Autoregression Model Identification Restrictions

This study uses multi –equation systems approaches to study the effects of monetary

and fiscal policies on price level determination. As explained in Chapter Three, both

structural vector autoregressions and structural vector error correction models will be

used with the theoretical properties of the monetarist, unpleasant monetarist and fiscal

theories of price level supplying the identifying restrictions.

Recall that the first order conditions of the household optimisation problem generate

the demand for goods, real balances, and government bonds. In that study, the

government policy is defined such that the government issues bonds and collects taxes to

pay off its existing debt. Therefore, under zero government expenditures, $g_t = 0$,

aggregate supply is equal to the household consumption expenditures: $y_t = c_t$. Following

the model developed in Chapter Two, we can define a macro model given monetary and

fiscal policies, output growth (equation (5``)), money demand (equation (6)) and the

budget constraint equation (equation (7)). As before R_t is the nominal interest rate;

$m_t = \dfrac{M_t}{P_t}$ is the real balances; $b_t = \dfrac{B_t}{P_t}$ is the real outstanding government debt and

$\pi_t = \dfrac{P_t}{P_{t-1}}$ is the inflation rate:

$$\frac{1}{R_t}\frac{y_{t+1}}{y_t} = \beta\frac{1}{\pi_t} \tag{5``}$$

$$m_t = y_t\left(\frac{R_t}{R_t - 1}\right) \tag{6}$$

$$b_t = -tx_t + b_{t-1}R_{t-1}\frac{1}{\pi_t} + m_{t-1}\frac{1}{\pi_t} - m_t \tag{7}$$

Note that under a constant output growth rate and perfect foresight equation (5``) gives the relation between today's interest rate and inflation rate.

Recall that given these relations, the determination of the price level depends on the joint monetary –fiscal policy. Therefore, it is possible to analyse inflation rate dynamics using a structural vector autoregression model by imposing Sims –Bernanke type restrictions supplied by the theoretical properties of monetarist theory, unpleasant monetarist theory and the fiscal theory of the price level.

Monetarist Theory

As discussed in Chapter Two the monetarist theory argues that in a Ricardian environment with active monetary –passive fiscal policy the price level is determined by the monetary authority. The monetary authority sets its policy variable and leaves the fiscal authority with the burden of the solvency of the government budget constraint. The passive fiscal authority sets its policy variable to pay off its debt.

Focusing on the contemporaneous relations this model can be defined by the following implicit functions, f^k, k =1,...4:

$y_t = \varepsilon_S$

$\pi_t = f^1(R_t) + \varepsilon_D$

$R_t = \varepsilon_{MP}$

$m_t = f^3 (R_t, y_t) + \varepsilon_{MD}$

$tx_t = f^4 (b_t) + \varepsilon_{FP}$

$b_t = f^5 (\pi_t, m_t) + \varepsilon_{BD}$

where, ε_S is the aggregate supply shock, ε_D is the aggregate demand shock, ε_{MP} is the monetary policy shock, ε_{FP} is the fiscal policy shock, ε_{MD} is the money demand shock and ε_{BD} is the bond demand shock.

This system of six equations and six variables, $X_t = (y_t, \pi_t, R_t, m_t, tx_t, b_t)$ can be analysed using a SVAR approach with the following constraints the theory imposes on the system:

The contemporaneous value of y_t is affected by no other variable

The contemporaneous value of π_t is affected by R_t

The contemporaneous value of R_t is affected by no other variable

The contemporaneous value of m_t is affected by y_t, R_t

The contemporaneous value of tx_t is affected by b_t, π_t, m_t, y_t, R_t

The contemporaneous value of b_t is affected by π_t, m_t, y_t, R_t

Unpleasant Monetarist Arithmetic

In Chapter Two, the unpleasant monetarist arithmetic is defined as a weak form of the FTPL since it is driven by a Ricardian policy with active fiscal and passive monetary policies. An active fiscal policy, defined as tx being an exogenous variable, will force the monetary authority to set interest rate such that the government budget constraint will be satisfied for every price level. Following the discussion in Chapter Two, this model can be defined by the following system of implicit equations:

$y_t = \varepsilon_S$

$\pi_t = f^{1\prime}(R_t) + \varepsilon_D$

$R_t = f^{2\prime}(y_t, tx_t) + \varepsilon_{MP}$

$$m_t = f^{3\prime}(R_t, y_t) + \epsilon_{MD}$$

$$tx_t = \epsilon_{FP}$$

$$b_t = f^{5\prime}(\pi_t, m_t) + \epsilon_{BD}$$

Hence, the unpleasant monetarist arithmetic can be analysed in a system of six

equations and six variables, $X_t = (y_t, \pi_t, R_t, m_t, tx_t, b_t)$ using an SVAR approach, with the

following constraints:

The contemporaneous value of y_t is affected by no other variable

The contemporaneous value of π_t is affected by R_t, y_t and tx_t

The contemporaneous value of R_t is affected by y_t and tx_t

The contemporaneous value of m_t is affected by y_t and R_t

The contemporaneous value of tx_t is affected by no other variable

The contemporaneous value of b_t is affected by π_t, m_t y_t, R_t and tx_t

Fiscal Theory of Price Level

The strong from FTPL is defined in a non –Ricardian environment which allows for

active fiscal and monetary policies. Under these policies, the price level is determined by

the fiscal policy variable. Guided by the theoretical discussions in Chapter Two, such a

model can be defined by the following system of six equations:

$$y_t = \epsilon_S$$

$$\pi_t = f^{1\prime\prime}(y_t, tx_t) + \epsilon_D$$

$$R_t = \epsilon_{MP}$$

$$m_t = f^{3\prime\prime}(R_t, y_t) + \epsilon_{MD}$$

$$tx_t = \epsilon_{FP}$$

$b_t = f^{5//}(\pi_t, m_t) + \varepsilon_{BD}$

Given these contemporaneous relations it is possible to define the FTPL in a six

equation –six variable system with the following constraints:

The contemporaneous value of y_t is affected by no other variable

The contemporaneous value of π_t is affected by y_t and tx_t

The contemporaneous value of R_t is affected by no other variable

The contemporaneous value of m_t is affected by y_t, R_t

The contemporaneous value of tx_t is affected by no other variable

The contemporaneous value of b_t is affected by π_t, m_t, y_t, R_t and tx

Vector Error Correction Model Identification Restrictions

Chapter Three discussed the importance of common trends on multi –equation

modelling. Due to previous econometric studies on the cointegration relationships

between the interest rate and the inflation rate (i.e., the Fisher Equation) and between real

balances, income and the interest rate (i.e, the money demand equation), we will also

consider structural vector error correction models (SVECM) in case we find evidence for

the existence of cointegration relations.

Note that the long run behaviour of our model can be defined by the following

implicit functions

$$\pi_t = g^1(R_t), \quad m_t = g^2(y_t, R_t), \quad b_t = g^3(R_t, \pi_t)$$

together with constant aggregate supply, $y_{t+1} = y_t$ and the respective monetary –fiscal

policy.

Monetarist Theory

Recall that in a Ricardian environment, when the government is following an active

monetary –passive fiscal policy, the price level is determined by the monetary authority.

We defined an active monetary policy as the monetary authority setting R_t at a

predetermined level R^*, $R_t = R_t^*$ and a passive fiscal policy as the fiscal authority setting

taxes given the outstanding government debt, b_t and real balances, m_t, $tx_t = g^4(\pi_t, R_t, y_t)$.

Therefore, in the long run, the monetarist model is defined by the following implicit

equations:

$$\pi_t = g^1(R_t), \ m_t = g^2(y_t, R_t), \ b_t = g^3(R_t, \pi_t), \ tx_t = g^4(\pi_t, R_t, y_t), \ R_t = R_t^*, y_{t-1} = y_t$$

If the variables in this system of equations are I(1) processes then the system implies

the existence of 4 cointegration relationships: interest rate and inflation rate (R and π);

interest rate, real money supply and income (R, m, y); government debt, interest rate and

inflation rate (b, R, π); tax receipts, inflation rate, interest rate and income (tx, π, R, y).

In a system of six variables (y, π, R, m, tx, b), the existence of four common trends

implies that the system is driven by four transitory shocks and two permanent shocks. We

define the permanent shocks as technology and monetary policy shocks, which we label

$$\varepsilon_t^p = (\varepsilon_t^{Tec}, \varepsilon_t^{MP})$$

The monetarist theory argues that monetary shocks have no long run effect on

income and that technology shocks are ineffective on the long run interest rate and the

inflation rate. Therefore, for the vector of variables, $X_t = (y_t, R_t, \pi_t, m_t, tx_t, b_t)$, the long

run structure of a Ricardian environment with an active monetary –passive fiscal policy

captured by the following matrix of restrictions:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & x \\ x & x \\ x & x \\ 0 & x \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \Pi_{21} & 1 \end{bmatrix}$$

where, x represents the unrestricted parameters and the interpretation of the matrix A was discussed previously in Chapter Three.

Fiscal Theory of Price Level

As opposed to the monetarist theory, the FTPL argues that in a non –Ricardian environment with active monetary and fiscal policies, the price level determined by the fiscal authority. In Chapter Two we explained that in a non –Ricardian environment it is possible for the monetary (fiscal) authority to set its policy variable independent of the other authority's actions, so that $R_t = R_t^*$ and $tx_t = tx^*$. Recall that, under these policies, the price level is determined from the demand side, by the solvency of the intertemporal budget constraint.

Therefore, the FTPL is defined by the following implicit equations of steady state variables:

$$\pi_t = g^1(y_t, tx_t), \quad m_t = g^2(y_t, R_t), \quad b_t = g^3(R_t, \pi_t), \quad tx = tx^*, \quad R_t = R_t^*, \quad y_{t+1} = y_t$$

With an active fiscal –monetary policy this system of equation exhibits three cointegration relationships if the six variables are I(1): inflation rate, income and tax revenue (π, y, tx); real money supply, interest rate and income (R, m, y); interest rate inflation rate and government debt (b, R, π). This system of six variables (y, π, R, m, tx,

b) with three common trends has three permanent shocks, which we name technology, monetary policy and fiscal policy shocks, $\varepsilon_t^P = (\varepsilon_t^{Tec}, \varepsilon_t^{MP}, \varepsilon_t^{FP})$.

Following KPSW, the FTPL imposes the following long run restrictions on the system of equations:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ x & 0 & x \\ 0 & 1 & 0 \\ x & x & 0 \\ 0 & 0 & 1 \\ 0 & x & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \Pi_{21} & 1 & 0 \\ \Pi_{31} & \Pi_{32} & 1 \end{bmatrix}$$

where x represent the unrestricted parameters

As defined in detail in Chapter Two, the FTPL argues that monetary policy has no effect on fiscal policy variables, income and the inflation rate. Fiscal policy also does not affect income or the monetary policy variables but does affect the inflation rate. Moreover, due to the independence of the authorities, the technology shocks do not affect the interest rate, government revenue and government debt.

CHAPTER 5. DATA

We used U.S. quarterly data for the period, 1959:1 –1998:4 for each of the following

series. Real Gross Domestic Product (GDP, chained 1996 dollars), GDP price deflator,

three month Treasury Bill rate (secondary market), M1 (seasonally adjusted), government

receipts and gross federal debt, which are used for income, price level, interest rate,

money supply, government revenue and outstanding government debt, respectively.

These series were obtained from the web pages of the Federal Reserve Bank of St Louis,

the Federal Reserve Board and Economagic. Following KPSW (1991) we calculated the

annual percentage inflation rate as: $\pi_t = 400*(\log(p_t) - \log(p_{t-1}))$. M1, government

receipts and government debt are divided by the price level to get real balances, real tax

revenue and the real debt.

To learn the processes that generated these realisations and make reasonable

predictions, we need to assume that at least part of the data generating process is stable

overtime. These stability assumptions are called stationarity conditions. Since a time

series data is regarded as a collection of random variables with a joint probability density

function (pdf) defining the structure of the process, stationarity conditions are actually

restrictions on the joint pdf's.

The data generating process X_t is strictly / strongly / completely stationary if the joint

probability distributions depends only on the time intervals separating the observations,

not on the date of the observations: $F(x_{t1},....,x_{tn}) = F(x_{t1+k},....,x_{tn+k})$. for all positive

integers n and integers k, $t_1,....,t_n$. A much less restricted definition is X_t being covariance

/second order / weakly stationary. X_t is covariance stationary if the marginal distributions

for $x_1, x_2,\ldots$ have a constant mean, a constant variance and the autocorrelation function depends only on the time lag –not on the starting point of the observations: $\mu_t = \mu$ and $\sigma_t^2 = \sigma^2$, $\rho_{x(t),x(t+k)} = \rho_k$ for all integers t and k.

If the original data have a non–constant mean or an increasing variance, a transformation of the data may solve the problem. Differencing the time series can often handle the problem of a non –constant mean, whereas, taking the logarithm can stabilize variability, simplify the structure of the model, change the shape of the trend line and change the distribution of the residuals.

Following the collections of our data, we checked for the stationarity properties of our series. Since the original series for income, real balances, real government revenue and real federal debt indicated time –dependent variance we worked with their logarithms.

y: log (GDP),   m: log (M1) –log (p),

tx: log (government receipts) –log (p)   b: log (federal debt) –log (p)

The basic statistics and the plots of the data are given in Table 5.1 and Figures 5.1

Table 5.1. U.S Data

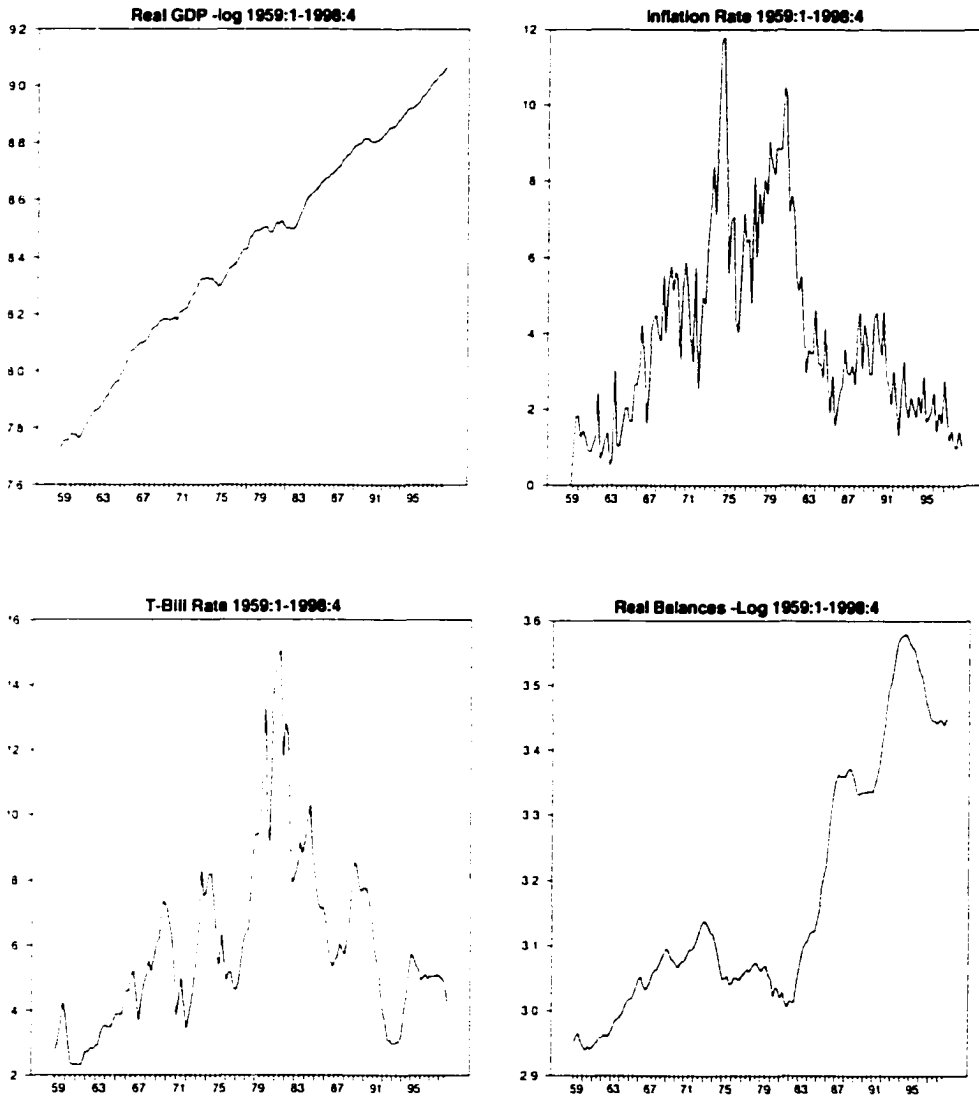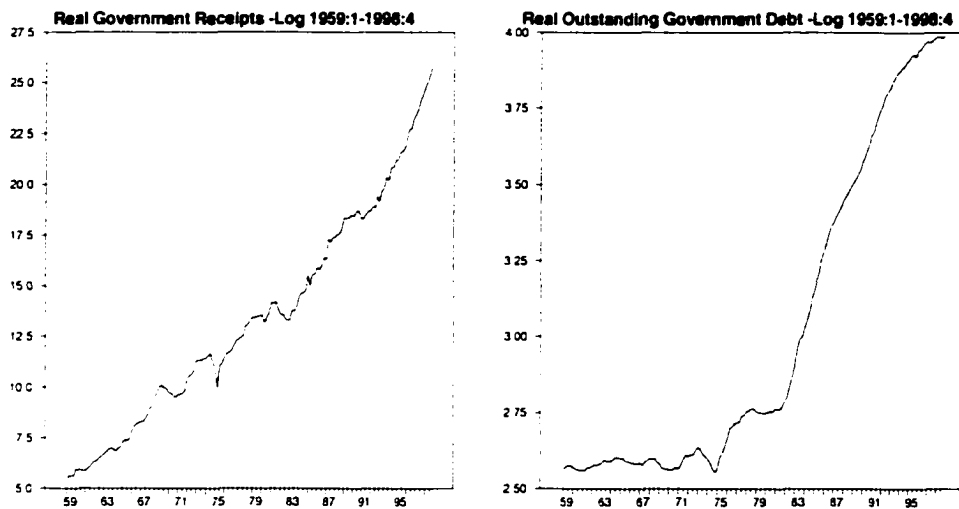| Variables	Mean	Std Error	Minimum	Maximum
y	8.434	0.371	7.728	9.067
π	3.918	2.515	0	11.795
R	5.956	2.652	2.303	15.053
m	3.171	0.191	2.939	3.580
tx	2.517	0.416	1.694	3.253
b	3.021	0.521	2.554	3.989

Figure 5.1. U.S Data

Figure 5.1. (continued)

The plots for y, m, tx and b indicate non-stationarity due to non constant means. For inflation and the interest rate the situation is not very clear. However, for all these series we followed the previous studies and regard them as nonstationary due to non –constant mean[56]. To capture the stationary component of these series we worked with first differences.

Table 5.2 summarizes the statistical properties of the differenced data series. Figures 5.2 graph these series. Note that, the graphs of the first differences appear to display stationary patterns fluctuating around their mean value zero with neither their mean nor their variance appearing to be time dependent.

Table 5.2. First Differences

Variables	Mean	Std Error	Minimum	Maximum
Dy	0.008	0.009	-0.020	0.037
Dπ	0.006	1.152	-3.519	3.336
DR	0.009	0.825	-3.736	4.459
Dm	0.003	0.011	-0.027	0.037
Dtx	0.009	0.019	-0.084	0.099
Db	0.008	0.013	-0.018	0.054

---

[56] See, for example, King, Robert G., Charles I. Plosser, James H. Stock, and Mark W. Watson (1991), Kyungho (2001)
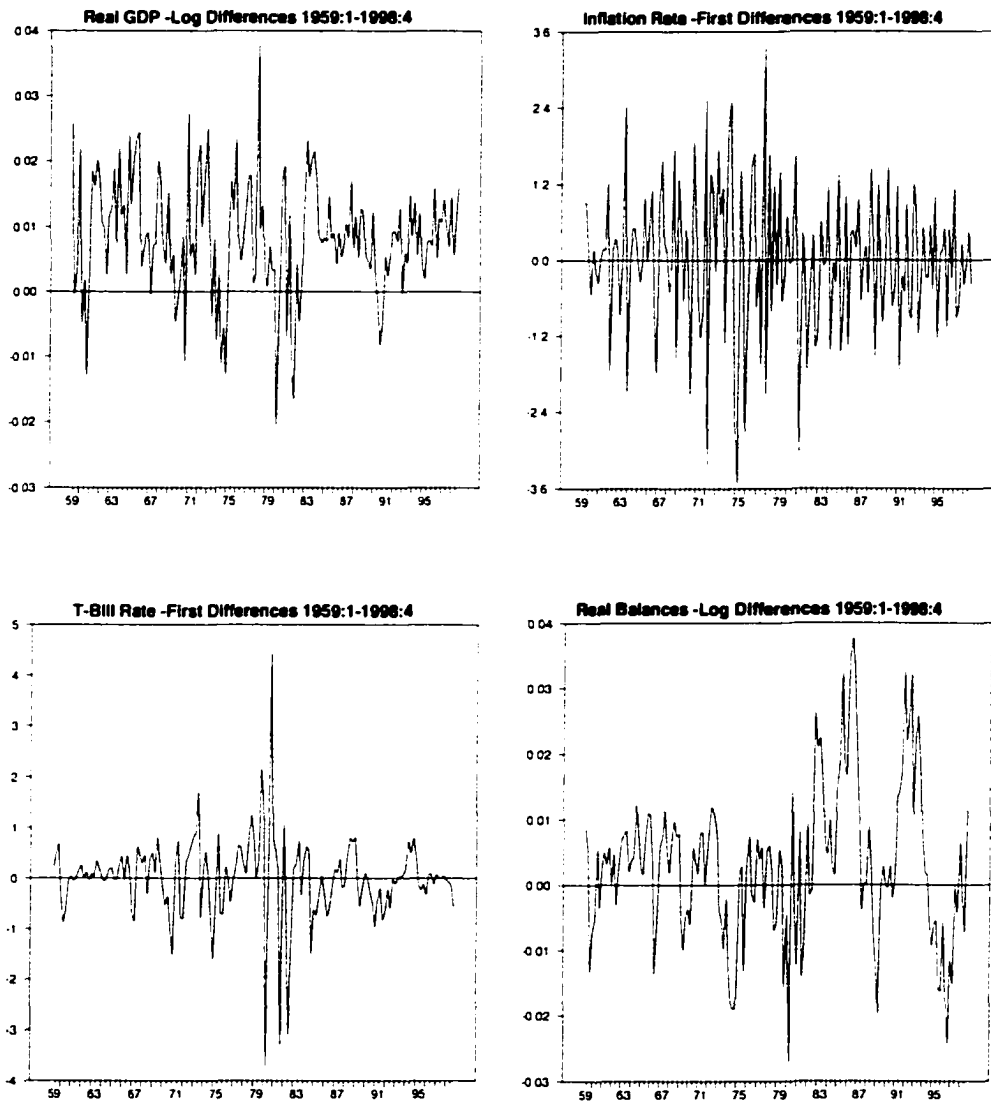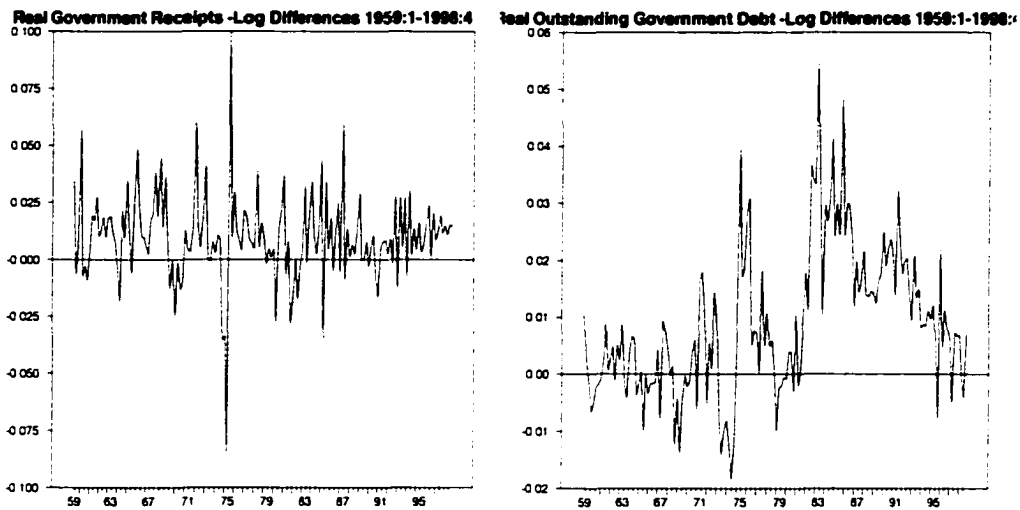
Figure 5.2. First Differences

Figure 5.2. (continued)

In general nonstationarity due to an increasing mean (which appears to be the case

for at least y, m, tx and b) may be due to a deterministic time trend or a stochastic trend,

which is expressed as a unit root process with drift. Even series without trend behaviour

(e.g., π and R) may be best characterized as unit root processes. Differencing the data

series to attain stationary behaviour is most appropriate for unit root processes [57]. To see

if our nonstationary series have unit roots, we conducted Dickey –Fuller (DF),

Augmented Dickey – Fuller (ADF) and Phillips –Perron unit root tests.

Unit Root Tests

Let {x_t} be an AR(1) process[58].

$$x_t = a_1 x_{t-1} + \varepsilon_t \qquad (22)$$

Assuming ε_t is white noise, the {x_t} sequence is covariance stationary if $|a_1|$ is less

than 1. In this case, we can derive the infinite –order moving average representation of x_t:

---

[57] Unit roots in the data generating process creates nonstationarity in VAR models.

[58] In time series models stationarity of a process is determined from the autoregressive part of the model.

$$x_t = (1 - a_1 L)^{-1} \varepsilon_t \tag{23}$$

$$x_t = a_1 \varepsilon_t + a_1^2 \varepsilon_{t-2} + \dots \tag{24}$$

Note that for $0 < |a_1| < 1$, the effect of a shock decreases overtime.

If in (22) $a_1 = 1$, so that the AR process has a unit root, the data is generated by a nonstationary process. In particular the model becomes, $x_t = \sum_{i=1}^{t} \varepsilon_i$ assuming $x_0 = 0$, and the variance becomes time dependent, increasing as t increases.

Under the null hypothesis of a unit root ($a_1 = 1$), the distribution of the t –ratio is non standard[59]. The Dickey –Fuller (DF), Augmented Dickey –Fuller (ADF) and Phillips – Perron (PP) unit root tests provide appropriate test procedures.

Dickey –Fuller Unit Root Test

In their Monte Carlo study. Dickey and Fuller (1979) provided the limiting distribution of the t statistic for the OLS estimator of the slope coefficient in (22) under the null of a unit root. They worked with three different autoregressive models of the data generating process and three null hypotheses. The models are:

$$\Delta x_t = q x_{t-1} + \varepsilon_t \tag{25}$$

$$\Delta x_t = a_0 + q x_{t-1} + \varepsilon_t \tag{26}$$

$$\Delta x_t = a_0 + q x_{t-1} + a_2 t + \varepsilon_t \tag{27}$$

---

[59] OLS models hypothesis testing requires the variables to be stationary and thus the error terms representing the deviations from the model decay overtime.

where, $\Delta x_t = x_t - x_{t-1}$ and $q = a_1 - 1$.[60] The corresponding null hypotheses are:

$H_0: q = 0$ (equation 25), $H_0: q = 0$ (equation 26) and $H_0: q = 0, a_2 = 0$ (equation 27). In this

setting testing for the existence of a unit root ($a_1 = 1$) is equivalent to testing if $H_0: q = 0$.

Let τ, τ_μ, τ_τ represent the t statistics for OLS slope coefficients from estimating

equations (25), (26) and (27) respectively. Dickey and Fuller (1979) derived the limiting

distributions these statistics under the null, which can be used to derive critical regions for

the one –sided test of H_0 against the alternative $q < 0$ ($a_1 < 1$).

We applied the Dickey –Fuller test to our data series. The calculated τ, τ_μ and τ_τ

statistics for the variables y, π, R, m. tx and b indicate the existence of a unit root in each

of these series.

For the models with / without constant and with trend we failed to reject the null

hypothesis of unit root at 1%, 5% and 10% significance levels for all series[61]. Similar

results are obtained for the model with a constant but with no trend term for the series y,

R, m, tx and b. Although, the τ_μ statistics for π are greater than the critical values –2.58

(10% level) and –2.89 (5% level), we fail to reject the null hypothesis of unit root at the

1% significance level.

---

[60] From equation 1:

$$x_t = a_1 x_{t-1} + \varepsilon_t$$
$$x_t - x_{t-1} = a_1 x_{t-1} - x_{t-1} + \varepsilon_t$$
$$\Delta x_t = (a_1 - 1)x_{t-1} + \varepsilon_t$$

[61] Empirical Cumulative Distribution:

Sample size:100	0.01	0.05	0.10
τ	-2.60	-1.95	-1.61
τ_μ	-3.51	-2.89	-2.58
τ_τ	-4.04	-3.45	-3.15

Table 5.3. Dickey –Fuller Unit Root Tests

	y	π	R	m	tx	b
τ	11.49	-1.54	-0.71	3.34	5.95	9.55
τ_μ	-1.31	-3.05	-2.11	0.13	-0.96	4.38
τ_τ	-2.16	-3.08	-1.95	-0.99	-2.69	-2.19

Augmented Dickey –Fuller Unit Root Test

To allow for serially correlated changes, consider the following extensions of equations (25)–(27):

$$\Delta x_t = q x_{t-1} + \sum_{i=2}^{k} \beta_i \Delta x_{t-i+1} + \varepsilon_t \tag{28}$$

$$\Delta x_t = a_0 + q x_{t-1} + \sum_{i=2}^{k} \beta_i \Delta x_{t-i+1} + \varepsilon_t \tag{29}$$

$$\Delta x_t = a_0 + q x_{t-1} + a_2 t + \sum_{i=2}^{k} \beta_i \Delta x_{t-i+1} + \varepsilon_t \tag{30}$$

where, as before q = 0 under the unit root null hypothesis. Under the alternative hypothesis. q < 0 and equations (28) –(30) can be rewritten to express x_t as a (trend) stationary k –th order autoregressive process. Dickey and Fuller showed that the limiting null distributions of the t –ratios for q derived from the OLS estimates of (28) –(30) do not depend on k and are equivalent to the Dickey –Fuller distributions associated with the t –ratios for q in (25) –(27). Hence, the Augmented Dickey –Fuller (ADF) unit root test is actually the modification of the DF test allowing for (k) lagged levels instead of 1.

Since the distribution is the same as before, the ADF tests use the same critical values as the DF tests. Table 5.4 shows the calculated ADF statistics for a model without

deterministic parts and a lag length determined by the AIC model selection criteria[62].

Given the lag length, the calculated ADF statistics for all models fail to reject the null

hypothesis of a unit root for; y, m, tx and b at the 10% significance level and for R for the

model with constant and no trend at the 5% significance level.

Table 5.4. Augmented Dickey –Fuller Unit Root Test

Variable	y	π	R	m	tx	b
Lag length	1	2	7	1	0	2
No constant + No trend	6.36	-0.94	-0.58	1.39	5.96	2.26
Lag length	2	1	5	1	0	2
Constant + no trend	-0.87	-2.35	-2.86	-0.51	-0.96	0.78
Constant + trend	-2.77	-2.39	-2.78	-1.96	-2.69	-1.91

Phillips –Perron Unit Root Test

The Phillips –Perron unit root test is an extension of the DF test and an alternative to

the ADF test. It extends the DF test by allowing for serial correlation in (25) –(27) but, in

contrast to the ADF test, does not require us to parameterise the serial correlation. It also

allows for less restrictive assumptions on the error process, allowing the errors to be

weakly dependent (as opposed to stationary) and to have heterogenous distributions.

The Phillips –Perron test statistics are the OLS t –ratios for q from equations

(25) –(27), transformed to account for possible serial correlation and /or

heteroscedasticity in the error terms in these equations. Call these statistics τ, τ_μ^*, and

---

[62] For given lag length four the ADF test statistics for all series fail to reject the null hypothesis of a unit root

Augmented Dickey –Fuller Unit Root Test for a given Lag Length

Variable	y	π	R	m	tx	b
Lag length	4	4	4	4	4	4
No constant + trend	4.19	-0.98	-0.78	1.28	3.96	1.81
Constant + no trend	-0.70	-1.86	-2.15	-0.98	-0.55	0.19
Constant + trend	-2.66	-1.91	-2.04	-2.23	-2.50	-1.85

τ_τ^*. Under the unit root null, the limiting distributions of theses transformed –ratios are the corresponding Dickey –Fuller distributions.

$$x_t = a_0^* + a_1^* x_{t-1} + \eta_t$$

$$x_t = a_0^- + a_1^- x_{t-1} + a_2^-(t - T/2) + \eta_t$$

Our results are given in Table 5.5. As in the DF test, the calculated PP statistics for the model with trend fail to reject the null hypothesis of a unit root for all series at the 10% significance level. The results are same for all series –except inflation, for a model without trend. For inflation the PP statistics without trend reject the null hypothesis of a unit root at the 5% and 10% significance levels, but not at the 1% level.

Table 5.5. Phillips-Perron Unit Root Test

Variable	y	π	R	m	tx	b
$H_0: a_1^* = 1$	-1.32	-3.06	-2.11	0.14	-0.97	4.40
$H_0: \tilde{a}_1 = 1$	-2.18	-3.10	-1.96	-1.00	-2.71	-2.21

Cointegration Test

The tests we have applied concluded that the series we are working with have unit roots. In our analysis of multi –equation models we have argued that for the reliability of our results it is crucial to incorporate the cointegration relationships –if there are any, and work with VECM instead of VAR models. Previous econometric studies and the economic theory we developed earlier imply the existence of some common trends in the system of six variables we are working with. Consequently, we applied Engle – Granger, Phillips – Ouliaris and Johansen cointegration tests to search the data for cointegration relationships.

Engle –Granger and Phillips –Ouliaris Cointegration Tests

These are both single –equation residual based cointegration tests. First ordinary

least squares is applied to estimate the assumed long run relationship among the variables.

Then. a unit root test is applied to the residuals to test the null hypothesis of no

cointegration. The Engle –Granger methodology uses a DF unit root test whereas the

Phillips –Ouliaris methodology applies a PP unit root test in this two –step procedure.

However. the critical values for these tests are not the same as the critical values used in

standard DF and PP unit root tests because the tests are being applied to the residuals

rather than the unobservable cointegration errors. One important difference between these

two methods is the issue of invariance with respect to the selection of the regression's

dependent variable. In small sample the Engle –Granger test results are sensitive to the

choice of the dependent variable. In contrast, the Phillips –Ouliaris multivariate trace test

is invariant to the normalization of the cointegration relation. Below we describe these

tests in more detail.

For a given regression equation.

$$w_t = \beta_0 + \beta_1 z_t + e_t \tag{33}$$

where, z_t can be vector –process. If the variables w_t and z_t are cointegrated of order (1,1)

(CI (1,1)) the deviations from their long run relation will go back to their mean level. That

is, the error process in (33) is stationary.

The model is fit by OLS and a unit root test is applied to the residuals { $\hat{e}_t$ }. which

are modelled as:

$$\Delta \hat{e}_t = q_1 \hat{e}_{t-1} + v_t \tag{34}$$

The Engle –Granger Methodology uses DF statistics to test the null hypothesis of a unit root, i.e.. the null hypothesis of no cointegration, H_0: $q_1 = 0$. If we cannot reject the null hypothesis, we conclude the residuals have a unit root and thus, the variables in question are not cointegrated. In contrast, if the unit root is rejected, we conclude that w_t and z_t are cointegrated of order (1.1).

Note that, if in (34) $\{ v_t \}$ is not a white noise process the ADF unit root test is used instead of the DF test. Thus, we estimate equation (35) instead of (34):

$$\Delta \hat{e}_t = q_1 \hat{e}_{t-1} + \sum_{i-1}^{l} q_{i-1} \Delta \hat{e}_{t-i} + v_t \qquad (35)$$

Since we are calculating the unit root test statistics from the estimated $\{ v_t \}$ sequence, rather than from the actual (unobserved) sequence, the Dickey –Fuller table of critical values cannot be used. For cointegration tests, the appropriate distributions are provided by Engel –Yoo (1987)[63].

The Phillips –Ouliaris multivariate trace statistics uses orthogonal regression, which is invariant to the normalization of the equation.

Let the data generating process $z_t = (w_t \ h_t')'$ be generated as:

$$z_t = \Psi z_{t-1} + \xi_t$$

where, ξ_t has zero mean and a finite variance.

The Phillips –Ouliaris multivariate trace, P_z statistics is[64]:

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No. of variables	1%	5%	10%
2	-4.07	-3.37	-3.03
3	-4.45	-3.93	-3.59
4	-4.75	-4.22	-3.89
5	-5.18	-4.58	-4.26

[64] In their paper Phillips and Ouliaris provides two more tests for cointegration. However, they do not have the property of invariance to order of variables in the equation

$$P_z = T * tr\left(\frac{T\hat{\Omega}}{\sum_{t=1}^{T} z_t z_t'}\right)$$

where, $t\hat{\Omega} = t\left(T^{-1}\sum_{t=1}^{t}\hat{\xi}_t\hat{\xi}_t + T^{-1}\sum_{s=1}^{t} j_{st} \sum_{t=s+1}^{T}\left(\hat{\xi}_t\hat{\xi}_{t-s}' + \hat{\xi}_{t-s}\hat{\xi}_t'\right)\right)$ is the estimate of the covariance

matrix of z_t and $\left(\frac{T}{\sum_{t=1}^{T} z_t z_t'}\right)^{-1}$ is the observed sample moment matrix[65].

Cointegration in the model will be reflected in the sample moment matrix and so in the P_z statistics.

Phillips –Ouliaris provides the asymptotic critical values for this test statistic. The null hypothesis of no cointegration is not rejected if the computed value of statistics is smaller than the appropriate critical value[66].

The first step in the Engle –Granger and Phillips –Ouliaris tests is to estimate the long run relationship among the variables. The regressions that we estimated are based on the steady state relationships defined by our economic theory. These relationships are:

1) $\pi_t = \alpha R_t + \varepsilon_t$

2) $m_t = \rho_y y + \rho_R R_t + \varepsilon_t$

3) $b_t = \beta_R R_t + \beta_\pi \pi_t + \varepsilon_t$

4) $tx_t = \gamma_\pi \pi_t + \gamma_R R_t + \gamma_y y_t + \varepsilon_t$

---

[65] $j_{st} = 1 - s/(l+1)$ is the lag window (Newey and West (1987))

[66] Phillips and Ouliaris (1990)

Right hand side var.	0.05	0.1	0.01
N=1	40.8	33.9	55.2
N=2	71.2	62.1	89.6
N=3	109.74	99.2	131.5

5) $\pi_t = \lambda_y y_t + \lambda_{tx} tx_t + \varepsilon_t$

Table 5.6 provides the test statistics calculated from the application of the Engle –

Granger and Phillips –Ouliaris methodologies to these cointegration relations.

The estimation results are as follows:

Cointegration relation between interest rate and inflation rate:

Recall that, for a separable utility function equation (5) is the Fisher equation,

defining a positive long run relationship between the nominal interest rate and the

inflation rate. According to the Fisher equation, in the long run the real interest rate is

independent of the inflation rate. Therefore, an increase in the inflation rate is fully offset

by an equal increase in the nominal interest rate. Our estimate of that long run

relationship

- $\pi_t = 0.22 + 0.62 R_t$

  (0.6)   (10.7)

indicates a positive relationship between the interest rate and the inflation rate, though

the coefficient is well less than one[67]. The Engle –Granger cointegration test using the DF

statistic rejects the null hypothesis of no cointegration at 1%, 5% and 10% significance

levels. The Engle –Granger cointegration test using the ADF statistic for models with

constant only rejects the no cointegration null at the 10% significance level with lag

length equal to 1. The calculated PO statistics reject the null hypothesis of no

cointegration at all standard significance levels. Therefore, the results from the EG and

PO residual –based cointegration tests suggests that, contrary to our theory, the interest

rate and the inflation rate are not cointegrated. This also means that there is not a stable

Fisher equation for the U.S economy over the sample period.

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[67] The values in parentheses are the t –values.

Table 5.6. Engel –Granger and Phillips –Ouliaris Methodology

Relation	1	2	3	4	5
DF	-4.15	-2.058	-1.65	-1.42	-3.01
ADF					
Lag 1	-3.22	-2.38	-1.93	-1.30	-2.31
Lag 4	-1.86	-2.26	-1.69	-0.87	-1.81
Lag 8	-1.39	-2.52	-1.66	-0.74	-1.77
PO	58.13	39.57	70.43	74.56	31.54

Cointegration relation between real balances, interest rate and income:

Households hold money for its purchasing power. Therefore, the demand for money is the demand for real balances. Economic theory argues for a positive (negative) relation between money demand and income (interest rate). Since the interest rate is the cost of holding money the trade off between cost and benefit of holding money imposes a decreasing money demand due to higher interest rates. However, an increase in income increases the demand for money for transaction purposes and so households demand more money as their income increases. The estimated money demand function is

- $m_t = -0.98 - 0.03R_t + 0.51y_t$
  (-9.1) (-16.1) (39.2)

Our estimation results reflect the behaviour economic theory indicates. However, in contrast to previous studies we could not find evidence for cointegration among these variables. The DF, ADF and PO tests do not reject the null hypothesis of no cointegration at all standard significance levels.

Cointegration relation between government debt, interest rate and inflation rate:

The demand for government bonds depends on its yield, the interest rate. A higher nominal interest rate (inflation rate) increases (decreases) the real interest rate, which

increases (decreases) the demand for government bonds. The estimated bond demand

function is:

- $b_t = 3.11 + 0.06R_t - 0.12\pi_t$

$\quad$ (33.7) (3.4) $\quad$ (-6.1)

The signs on the coefficients are consistent with the theory, although the coefficient

on the inflation rate is twice as large in absolute value as the coefficient on the interest

rate. Although the DF and ADF tests do not reject the null hypothesis, the PO

multivariate trace tests reject the null hypothesis of no cointegration at 1% and 5%

significance level.

Cointegration relation between government revenue, inflation rate, interest rate and

income:

The government sets its tax revenue to guarantee the solvency of the government

budget constraint. Recall that taxes are defined as units of consumption goods. Thus,

higher income, which is equivalent to an increase in the supply of goods in our pure

exchange economy, leads to higher tax revenue.

The Ricardian government is responsible for the solvency of its budget constraint,

$-\tau_t = m_t - \dfrac{1}{\pi_t}m_{t-1} + b_t - \dfrac{R_{t-1}}{\pi_t}b_{t-1}$. High inflation rate decreases the real value of

government debt, $\dfrac{1}{\pi_t}m_{t-1} + \dfrac{R_{t-1}}{\pi_t}b_{t-1}$ thus requires less tax revenue. Hence, taxes decrease

due to a higher (lower) inflation rate (interest rate)..Similarly, a higher interest rate lowers

money and bond seignorage, $b_t - \dfrac{R_{t-1}}{\pi_t}b_{t-1}$ and so, requires higher taxes. The estimated

relationship between tax revenue, the inflation rate, the interest rate, and income is

- $tx_t = -6.9 + 0.006\pi_t - 0.002R_t + 1.12y_t$

  $(149.3)\ (5.73)\quad (-1.54)\quad (198.5)$

The DF and ADF, tests reject the null hypothesis of no cointegration at the 5% significance level whereas the PO test rejects the null hypothesis of no cointegraion at all significance levels. However, note that the signs of the estimated coefficients on the inflation rate and the interest rate are incorrect.

Cointegration relation between inflation rate, income and government revenue:

It is assumed that households are rational and their current demands are determined by their current and expected futureincome. In a Ricardian environment, where the fiscal authority is bound by the solvency of the government budget constraint, a rational household knows that lower taxes today will cause higher taxes tomorrow and will leave the present value of its lifetime income unchanged. However, in a non –Ricardian environment, with active fiscal and monetary policies lower taxes today do not necessarily lead to higher taxes tomorrow. A rational household knowing that, will realize the increase in his lifetime income and will increase his demand which will in turn cause a higher inflation rate. Therefore, we expect that an increase in current taxes or a decrease in current income will decrease the inflation rate. Our estimated relationship is

- $\pi_t = 294.6 + 42.1tx_t - 47.02y_t$

  $(6.38)\quad (6.34)\quad (-6.32)$

For this long run relationship the DF and ADF tests do not reject the null hypothesis of no cointegration, whereas the PO test rejects the null hypothesis of no cointegration at all significance levels. Note that the signs of the estimated slope coefficient are inconsistent with our theory.

The estimated coinetgration relationships and the results of the residual –based test are not very good from our point of view. In most cases, the test results are either mixed or do not reject the no –cointegration null. And, in a number of cases, the point estimates of the slope coefficient are of the wrong sign.

The Johansen Methodology

In Chapter Three we explained that the rank of the matrix Γ in the following multi – equation system determines the number of cointegration relationships in the vector process X_t:

$$\Delta X_t = \Gamma X_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta X_{t-1} + e_t$$

As opposed to the previous cointegration tests, which were single equation, two –step estimation procedures, Johansen's (1990) algorithm uses the maximum likelihood estimation technique to estimate the matrix, Γ and its rank, leading to a test of the number of (linearly independent) cointegrating relationships and estimates of these relationships. The procedure is as follows:

First, we determine the lag length of the system using a model selection criteria such as AIC, SBC or the likelihood ratio test. Our results for the system of 6 equations y, π, R, m, tx, b indicated that the model with / without drift has lag length 4(1) according to AIC

(SBC)[68].

Then we estimated the selected model using maximum likelihood estimation based upon the Gaussian innovation process and found the characteristic roots, λ of the estimated matrix Γ.

Johansen formed two types of statistics to check for cointegration, λ_{max} and λ_{trace}. The λ_{max} statistic is used to test for the existence of r against the alternative of r+1 cointegration vectors:

H_0: r = 0, H_A: r = 1; H_0: r = 1, H_A: r = 2; H_0: r = 2, H_A: r = 3;.....

$$\lambda_{max} = -T\ln(1 - \lambda_{r+1})$$

where T = number of observations and λ_r denotes the r –th eigenvalue when the eigenvalues are arranged in descending order..m = number of characteristic roots

The λ_{trace} statistics are used to test if there are at most, r cointegration vectors. That is,

H_0: r = 0, H_A: r ≠ 0; H_0: r <= 1, H_A: r > 1; H_0: r <= 2, H_A: r > 2;.....

$$\lambda_{trace} = -T\sum_{i=r+1}^{m}\ln(1 - \lambda_i)$$

Note that to test for no cointegration relations among the set of variables we can use either or both of these statistics. The asymptotical null distribution of these statistics are provided by Johansen and Juselius (1990)

Lag Length	With constant (AIC / SBC)		Without constant (AIC / SBC)	
Lag 12	-5599.13	/ -4289.32	-5574.94	/ -4283.08
Lag 8	-5721.26	/ -4834.18	-5804.48	/ -5086.37
Lag 4	-5895.54	/ -5439.03	-5883.47	/ -5445.21
Lag 1	-5858.52	/ -5729.89	-5802.12	/ -5691.87

The results are reported in Table 5.7. For the model without drift (with drift) and

using a lag length one, our results indicate that there are 4 (3) cointegration relationships.

Table 5.7. Johansen Methodology

Lag 1 / no constant in the system

H_0: r	λ_{max}	λ_{trace}	crit. val. max	crit. val. trace
0	200.42	330.51	22.76	78.30
1	61.38	130.09	18.96	55.54
2	40.24	68.71	15.00	36.58
3	21.18	28.47	11.23	21.58
4	6.72	7.29	7.37	10.35
5	0.57	0.57	2.98	2.98

Lag 1 / constant in the system

H_0: r	λ_{max}	λ_{trace}	crit. val. max	crit. val. trace
0	152.11	264.32	24.63	89.37
1	62.44	112.21	20.90	64.74
2	30.76	49.77	17.14	43.84
3	12.00	19.01	13.39	26.70
4	6.66	7.01	10.60	13.31
5	0.35	0.35	2.71	2.71

As described in previous sub –section, our theory implies that there are five

cointegration relationships among the variables in our system. The Johansen tests indicate

that there are at most four and as few as three cointegration relationships. Thus, these

results are not fully supportive of our theory.

Conclusion

Chapter Four indicated the importance of whether there are cointegration relations in

a multi –equation systems analysis. Therefore, in Chapter Five we tested the long run

relations we suspect. The number of cointegration relationships we found varied across

the different tests we applied. In addition, in a number of cases the estimated coefficients

in the cointegrating relations were of wrong sign. It is possible that the differences in

results and the sign inconsistencies may be explained by the inadequacy of the sample

size or the low test power rather than by inadequancies of the theory. However, we

proceed with both a structural VAR model and structural VECM in the next chapter.

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CHAPTER 6. ESTIMATION RESULTS

In this chapter, we report and analyse the results from estimating the multivariate

models that were introduced in previous chapters. First, we modelled the U.S. economy

using the structural vector autoregressions (SVAR) described in Chapter Four for each of

the competing theories: monetarist theory, unpleasant monetarist arithmetic and the

FTPL. First differences of y, π, R, m, tx and b were fit by FIML to a first –order system

(based upon AIC and SBC lag length tests) with an intercept in each equation. We

estimated the system by FIML since our models are over –identified.[69] [70] Second, we

incorporated the cointegration relationships defined under the monetarist theory and

FTPL and modelled the economy as a structural VECM (SVECM) with lag length one

and an intercept in each equation, following the estimation procedure described in

Chapter Four.

The estimation results, which are summarized by the impulse response functions and

variance decomposition tables, are given in Appendix C (SVAR) and Appendix D

(SVECM). We discuss these results below.

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[69] System is over -identified if there are more than $(n^2-n)/2$ restrictions
[70]

Lag Length	With constant (AIC/SBC)	Without constant (AIC/SBC)
Lag 12	-5470.2 / -4163.4	-5446.3 / -4139.4
Lag 8	-5632.2 / -4747.1	-5610.6 / -4725.5
Lag 4	-5803.8 / -5348.2	-5773.5 / -5317.9
Lag 1	-5879.1 / -5750.7	-5838.2 / -5709.8

Structural Vector Autoregression Model Estimation Results

Before looking at the impulse responses and the variance decomposition tables, we

applied Granger causality tests to check for the explanatory power of a lagged variable in

a regression. The stochastic process ξ_t "Granger causes" the process h_t if in the regression

of h_t on its own past history and the past history of ξ_t, the past history of ξ_t is irrelevant.

That is, ξ_t Granger causes h_t if in the regression:

$$h_t = \xi_{10} + \xi_{11}(1)s_{t-1} + \xi_{11}(2)s_{t-2} + \xi_{11}(3)s_{t-3} + \ldots + \xi_{12}(1)h_{t-1} + \xi_{12}(2)h_{t-2} + \ldots + e_{ht}$$

at least some of the coefficients, $\xi_{11}(1), \xi_{11}(2), \xi_{11}(3) \ldots$, are significantly different from

zero. The null hypothesis that ξ_t does not Granger cause h_t can be tested by a standard F –

test of the null hypothesis, $H_0 : \xi_{11}(1) = \xi_{11}(2) = \xi_{11}(3) = \ldots = 0$.

This idea can be extended directly to the case where there are additional lagged

explanatory variables in the regression, which is the form of the test that we apply. That

is, we regress each dependent variable (in differenced form) on its own lagged values and

the lagged values of the other five variables in the system. For each equation we apply the

F –test to the lagged values of each of the six variables (i.e., the own lagged values and

the lagged values of the other five variables). The results are reported below in Table 6.1.

Using a 10 percent significance level, the Granger causality test results indicate that

each variable causes itself. In addition, y granger causes m and tx; R Granger causes all

variables except y; m Granger causes y, π and R; tx Granger causes m and b Granger

causes π, R and m.

Table 6.1. Granger – Causality Test Results*:

Dependent Variable / Lagged Variable	y	π	R	m	tx	b
y	7.01	0.7	2.5	3.6	10.6	0.5
	(0.08)	(0.39)	(0.11)	(0.06)	(0.00)	(0.45)
π	0.2	14.5	0.04	1.2	0.6	0.05
	(0.64)	(0.00)	(0.84)	(0.28)	(0.43)	(0.81)
R	0.9	4.9	2.04	48.7	2.7	3.6
	(0.31)	(0.03)	(0.01)	(0.00)	(0.10)	(0.06)
m	3.4	7.2	4.08	106.6	2.4	0.08
	(0.06)	(0.01)	(0.04)	(0.00)	(0.11)	(0.76)
tx	0.5	0.5	0.2	4.07	9.8	0.9
	(0.45)	(0.49)	(0.61)	(0.04)	(0.002)	(0.32)
b	0.5	3.6	2.7	5.6	0.6	103.3
	(0.47)	(0.06)	(0.09)	(0.02)	(0.44)	(0.0)

*Values in parenthesis are p –values

These results are conflicting with our assumptions. We assumed an exogenous y. However, according to Granger causality test results, real GDP is a function of real balances and itself. Also, the results indicate that government debt is only a function of itself and interest rate, not inflation rate as we assumed. Moreover, Granger causality indicates that real balances is a function of taxes and real outstanding government debt as well as y and R.

Monetarist Theory

The impulse responses of our model for the monetarist theory indicate that inflation rate is affected by the aggregate demand shock as well as monetary policy shocks. According to our impulse responses the disinflationary effect of a positive aggregate supply shock is confronted by increased tax revenue, money and bond demand[71]. Thus,

---

[71] Recall that real balances are positively related with income and government bond is a function of real balances. Thus, higher income leads to higher money demand and therefore higher demand for government bonds.

inflation rate being totally determined by the independent monetary authority is not

affected by the aggregate supply shock contemporaneously.

Responses to Supply Shock
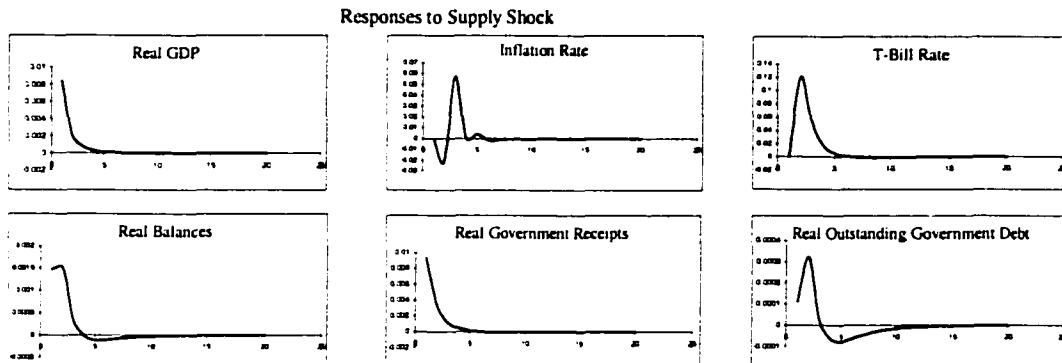
Aggregate supply / demand analysis argues that given aggregate supply a positive

aggregate demand shock increases inflation rate in the short run. Our impulse responses

indicate this same relation. Interest rate (being determined by the monetary authority

exogenously) and the real balances (being a function of income and interest rate only) are

not affected by the demand shock contemporaneously. However, higher inflation rate

decreases the real rate of interest and so the demand for government bonds. To overcome

the resulting decrease in bond seignorage government increases taxes.

Responses to Demand Shock
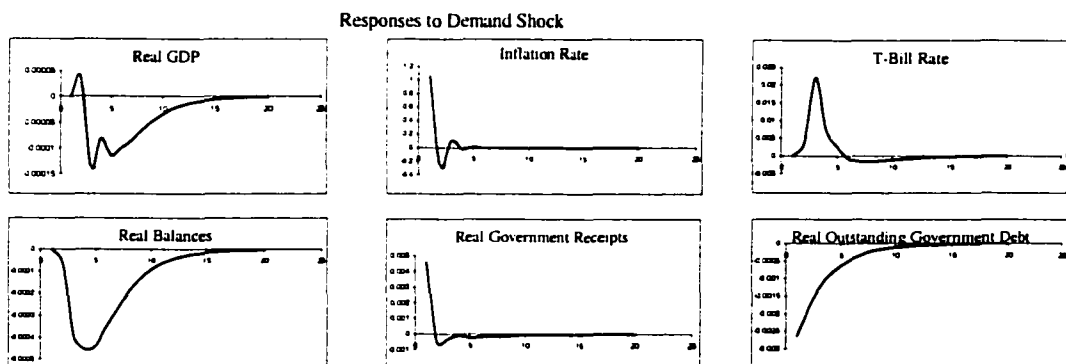
The impulse responses indicate that the fiscal policy variable does not have any

contemporaneous effect on any of the variables. All variables react to this shock with one

period lag. Aggregate supply, interest rate, real balances and government bonds decrease

before going back to their steady state values. As opposed to their u –shaped impulse

response curves a positive fiscal policy shock creates oscillations in inflation rate.

Moreover, the variance decomposition table indicates the low explanatory power of the

fiscal policy variable on inflation rate. Fiscal policy explains only 0.2% (0.4%) of

variation in inflation rate in the second (third) period.

Responses to Fiscal Policy Shock
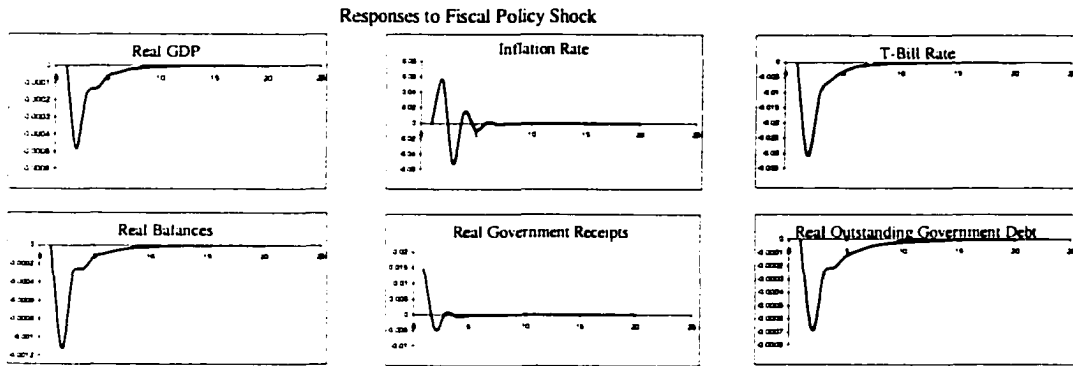
As the monetarist theory argues, our impulse responses indicate that the monetary

authority controls the rate of inflation. An increase in interest rates increases inflation rate

but decreases real balances, government bonds and t period real wealth w_t.[72]

Responses to Monetary Policy Shock
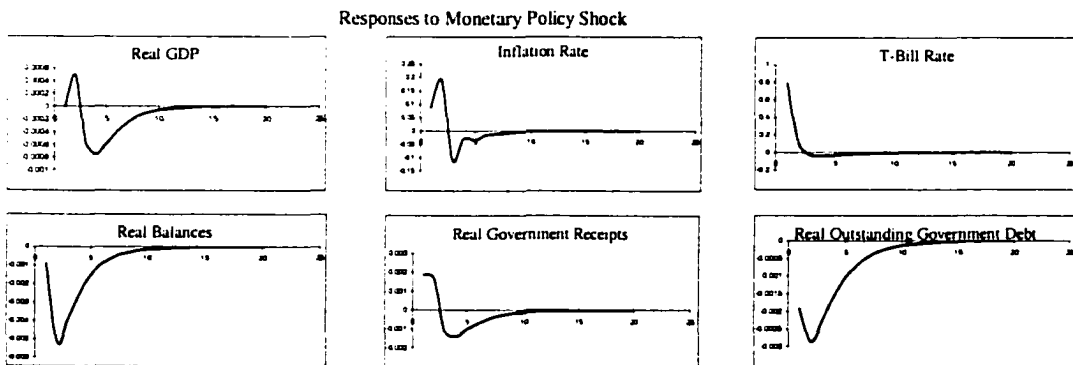
Recall, that under monetarist theory tax revenue is determined by the fiscal authority

to satisfy the solvency of the government budget constraint. Thus, tax revenue should be

such that the real wealth is equal to the government revenue $(m_t + b_t + \tau_t)$. Our impulse

responses indicate that the decrease in real wealth is less than the decrease in real

balances and real debt. Therefore, taxes increase as a response to higher interest rate, to

---

[72] Period t real wealth is: $w_t = \dfrac{W_t}{P_t} = m_{t-1}\dfrac{1}{\pi_t} + R_{t-1}\dfrac{1}{\pi_t}b_{t-1}$.

satisfy the solvency of the government budget constraint. However, monetary policy is not as effective on inflation rate as the monetarist theory argues. Although R, m and b granger causes inflation rate, inflation rate explains 99.3% (91.4%) of variation in itself in the first (third) period. Monetary policy explains only 0.6% of variation in inflation rate in first period and 3.4% (4.2%) of variation in the second (third) period.

Note that money supply, an endogenous variable of the model affect inflation rate only with a lag. Interest rate explains 1.6% (34.2%) of variation in m in the first (third) period and real balances explain 2.1% (2.3%) of variation in inflation rate in the third (fifth) period.

Unpleasant Monetarist Arithmetic

Contrary to the monetarist theory positive aggregate supply shock decreases (increases) inflation rate (interest rate) contemporaneously. The impulse responses of the monetarist theory indicated that inflation rate do not change in response to an aggregate supply shock. However, under the assumptions of the unpleasant monetarist arithmetic a positive aggregate supply shock does not create an equally effective demand movement. Thus, inflation rate decreases. Under the assumptions of the monetarist theory part of the increase in income was melted by increase in taxes. However, under an active fiscal policy taxes do not respond and a positive supply shock creates an increase in income out of taxes. Although, higher interest rate has a negative affect on real balances the impulse responses indicate that the effect of income on real balances is higher than the effect of interest rate.

Responses to Supply Shock
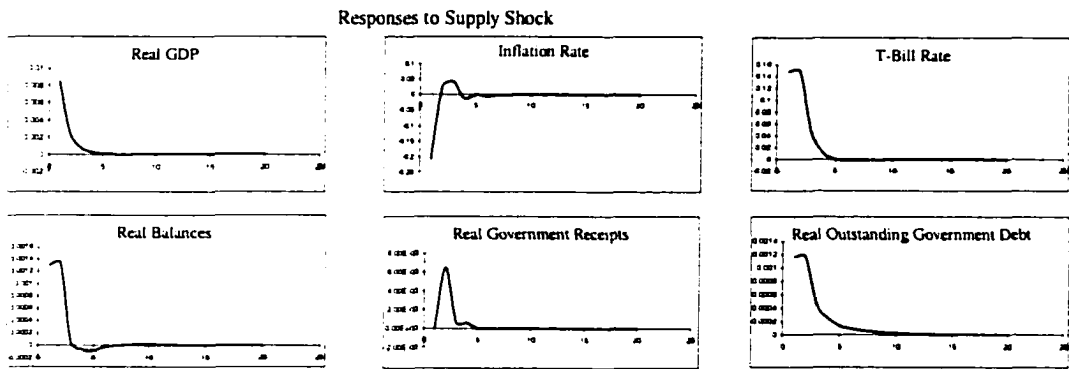
According to the unpleasant monetarist arithmetic, fiscal authority determines its

policy variable independent of the monetary policy and although monetary authority

determines the rate of inflation it is dependent on the fiscal authorities actions in order to

satisfy the government budget constraint. Our impulse responses indicate that the fiscal

policy variable is independent of all shocks except its own, contemporaneously. However,

monetary policy variable reacts to shocks to satisfy the government budget constraint. In

contrast to our results for the monetarist theory, interest rates increase as a response to a

positive aggregate supply and a positive fiscal policy shock.

Responses to Fiscal Policy Shock
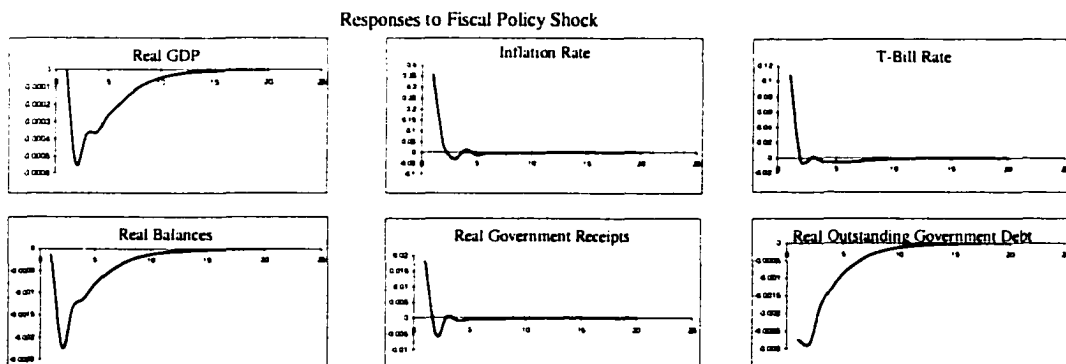
The impulse responses of a fiscal policy shock indicate the positive relation

between monetary and fiscal policy variables, as expected. A negative fiscal policy shock

increases the government deficit ($g_t - tx_t$). Monetary authority being responsible for the

solvency of the government budget constraint lowers interest rates to provide enough

money / bond seignorage (low interest rate increases money demand and so, increases

money seignorage for the government).

Unpleasant monetarist arithmetic argues that fiscal policy effects inflation rate only

through its effect on monetary policy variable. According to the variance decomposition
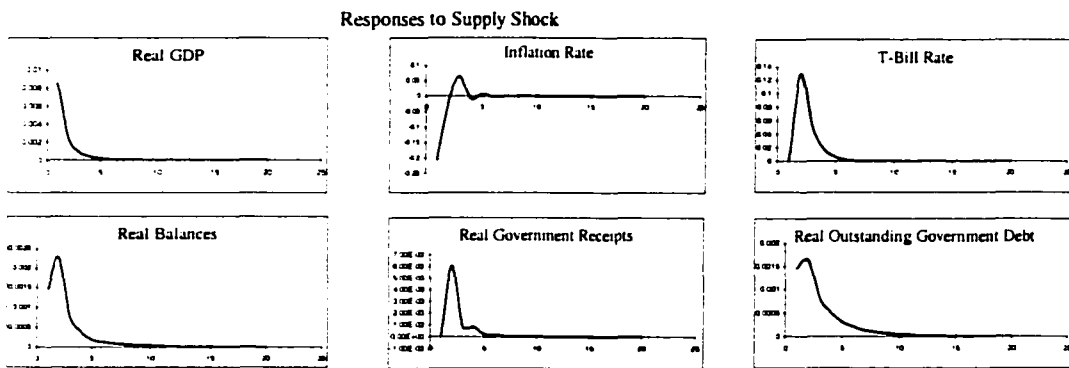
Responses to Monetary Policy Shock
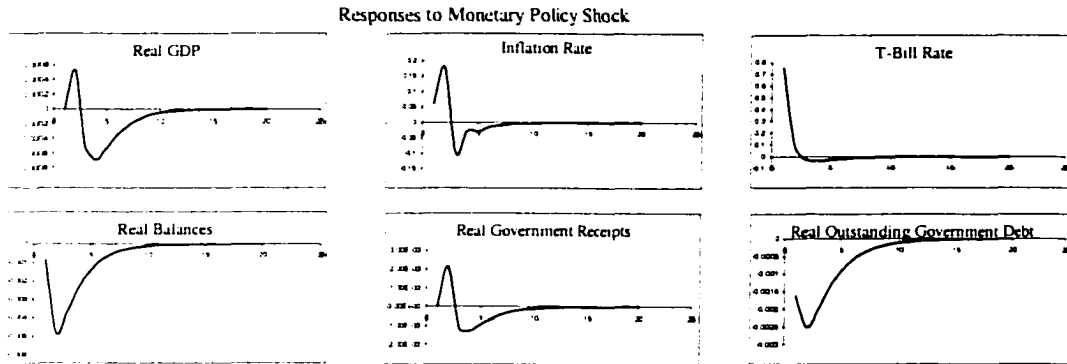
tables. taxes explain 10.5% (9.3%). whereas interest rate explains only 0.3% (3.2%)of

variation in inflation rate in the first (third) period (which is less than the explanatory

power of aggregate supply. (Aggregate supply explains 3.6% (3.3%) of variation in

inflation in the first (third) period). However, the fiscal policy variable taxes explain only

1.9% (1.75%) of variation in interest rates in the first (fifth) period, which is less than the

explanatory power of income and real balances. (Income explains 3.6% (6.9%) of

variation in inflation in the first (fifth) period, whereas real balances explain zero percent

(2.0%) of variation in inflation rate in the first (fifth) period.)

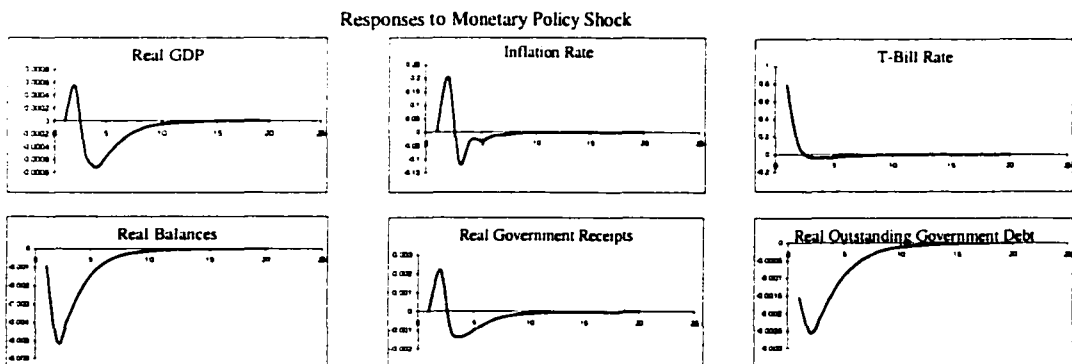Fiscal Theory of Price Level

Fiscal theory of price level assumes that both the monetary and the fiscal authority

act independently and the price level is determined by the fiscal authority as a solvency

condition of the government budget constraint. According to the impulse responses both

the fiscal and the monetary policy variables respond contemporaneously to their own

shocks. Inflation rate is affected by aggregate supply, demand and fiscal policy shocks contemporaneously.

According to the impulse responses a positive aggregate supply shock decreases inflation rate and increases money and bond demand. Neither the fiscal policy variable (as was the case in monetarist theory results) nor the monetary policy variable (as the case in unpleasant monetarist theory results) reacts to this exogenous shock contemporaneously.

Responses to Supply Shock

The impulse responses of inflation rate due to a positive monetary policy shock are very similar to the ones for monetarist theory and the unpleasant monetarist arithmetic.

Responses to Monetary Policy Shock

According to the impulse responses of the monetarist theory a positive monetary policy shock increases taxes. However, the impulse responses of the unpleasant monetarist arithmetic and the fiscal theory of price level indicate no contemporaneous changes in the fiscal policy variable. Under the assumptions of the unpleasant monetarist theory higher inflation rate decreases real wealth and increased money and bond demand absorbs the effects of the monetary policy shock. In case of FTPL the monetary policy

variable does not affect inflation rate contemporaneously. Recall that, according to the assumptions of FTPL government bonds are net wealth. Thus, price level is determined by the fiscal policy through this wealth effect. Since, wealth is defined as, $W_{t+1} = M_t + R_t B_t$, an increase in today's interest rate effects not today's but tomorrow's wealth and the price level.[73]

The variance decomposition tables of the FTPL indicate that in the first period inflation rate is affected by itself, the fiscal policy variable and income. Similar to our results for the unpleasant monetarist arithmetic model, taxes explain 10.5% (9.1%) of variation in inflation rate in the first (third) period. Although the interest rates and the real balances have no explanatory power on the contemporaneous variation in inflation rate, the monetary policy variable explains 3.8% (3.9%) of variation in inflation rate in the third (fifth) period. Note that the explanatory power of monetary policy shocks on the variation of inflation rate is almost same on all three models.

However, theory does not explain our results for the fiscal policy shock. FTPL argues that once the fiscal authority decreases taxes, households –believing that today's tax policy will not lead to higher taxes tomorrow, would like to consume more. Thus, inflation rate will increase. This negative relation between inflation rate and taxes is not seen in our impulse response graphs. Our results show that a positive fiscal shock increases inflation rate and decreases government debt.

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[73] Real wealth is: $w_{t+1} = \dfrac{W_{t+1}}{p_{t+1}} = m_t \dfrac{1}{\pi_{t+1}} + R_t \dfrac{1}{\pi_{t+1}} b_t.$

Therefore the household budget constraint is: $m_t + b_t + \tau_t = w_t$

Vector Error Correction Model Estimation Results

Monetarist Theory

The structural VECM identified by the assumptions of the monetarist theory, defines technology and monetary policy shocks.

The point estimates of impulse responses of variables due to a positive monetary policy shock are compatible with the theory. An increase in interest rates increases inflation rate but decreases real balances and taxes. This positive relation between inflation rate and the interest rate is seen in all impulse responses.

The impulse responses of variables in differences / levels indicate that a positive technology shock decreases inflation rate and interest rate, but increases output, taxes and government bonds. According to the monetarist theory interest rate, the monetary policy variable is determined independently by the monetary policy variable. Therefore, interest rates should not react to technology shocks contemporaneously.

Moreover, the impulse responses of the levels / differences indicate that aggregate supply, interest rate and inflation rate increases due to a positive fiscal policy shock. This is the relation we would expect from a model defined under the assumptions of the unpleasant monetarist arithmetic not a monetarist theory.

According to our variance decomposition tables, technology and monetary policy shocks explain 54% (57%) of variation in inflation rate in the first (fifth) period. However, most of this effect of permanent shocks on inflation rate is due to technology shocks. Contrary to the monetarist theory, monetary policy explains only % 0.01 (% 0.1) of variation in inflation rate in the first (fifth) period. The 53% and 12% of variation in inflation rate is explained by technology and fiscal policy shocks respectively. As for the

income, the variance decomposition table indicates that taxes explain 45% and interest rate explain 24% of variation in income.

The variance decomposition table indicates a passive monetary policy. Inflation (technology) explains 53% (30%) variation in interest rate in the first period, fiscal policy shock which is supposed to be ineffective on monetary policy variable (according to the monetarist theory) explains 16% (32%) of variation in interest rate in the first (fifth) period. (In the fifth period monetary policy (technology) shocks explain 0.1% and technology shocks explain 36% of variation in interest rate).

Fiscal Theory of Price Level

According to the fiscal theory of price level lower taxes leads to higher inflation rate. People believing that lower taxes today will not lead to higher taxes tomorrow, realizes an increase in their income due to this fiscal policy and demand more goods. Hence, given aggregate supply an increase in demand increases inflation rate. Our impulse responses of variables due to a fiscal policy shock certify this negative relation between taxes and inflation rate. The impulse responses of levels / differences indicate that income, inflation and interest rate increase (real balances and government bonds decrease) due to a negative fiscal policy shock –higher taxes.

Besides fiscal policy shock our FTPL model defined technology and monetary policy shocks. According to the impulse responses a positive technology shock leads to higher income and lower inflation and interest rate as expected. However, our results indicate a negative relationship between income and real balances and between income and taxes, which the model cannot explain. Similarly, for M1 used as monetary

aggregate, the impulse responses of the monetary policy shock indicate a positive relationship between interest rate and real balances[74].

According to the impulse responses, an increase in interest rates increases demand for money and government bonds and decreases inflation rate. The negative relation between the nominal interest rate and inflation rate is not what the fisher equation indicates.

Besides these problematic relations driven by the impulse response analysis, the variance decomposition tables contradict the arguments of FTPL too. Taxes explain 38%; income explains 19% whereas interest rate explains 1.2% of variation in inflation rate for the first period. For the independency of the monetary and fiscal policy variables, the variance decomposition table indicates that the monetary policy variable explain 3.9% and inflation rate explain 51% of variation in the fiscal policy variable. To sum, permanent shocks explain 24% of whereas, transitory shocks explain 75% of variation in taxes. Moreover. 24%, 26% and 1% of variation in interest rate is explained by income, interest rate and taxes respectively. The low explanatory power of taxes (interest rate) on monetary (fiscal) policy variable is what we expect. However, the significant power of government debt on interest rate is not what the theory predicts. According to the variance decomposition tables, government debt explains 40% of variation in interest rate.

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[74] When M2 is used as monetary aggregate, the estimation results for the relation between m, y and R indicate a positive relation between real balances and interest rate and between real balances and income.

$$m_t = -2.78 + 0.84 y_t + 0.003 R_t$$
$$\quad (0.08) \quad (0.01) \quad (0.001)$$

CHAPTER 7. CONCLUSION

In that study, we analysed U.S inflation rate using structural vector autoregression and structural vector error correction models under the assumptions of monetarist theory, unpleasant monetarist arithmetic and the fiscal theory of price level. Our aim was to distinguish the effects of monetary and fiscal policies on price level determination. We assumed taxes the fiscal policy variable and –in order to make all theories compatible with each other– interest rate is the monetary policy variable.

First, we worked with a structural vector autoregression model. Our results indicate the importance of monetary policy variables on price level determination. However, our variance decomposition tables and impulse responses provided evidence on fiscal policy effecting inflation rate through the monetary policy variable.

Since, previous studies provided evidence on cointegration relations between interest rate and inflation rate; money supply, interest rate and income, we proceeded with a structural vector error correction model. However, neither SVAR nor SVECM results provided any evidence of FTPL

Our results are based on the assumption that the monetary authority sets interest rate rather than money supply. Although, in today's world this is a realistic assumption, to be compatible with theoretical studies, a further study would be to test if our results hold for money supply targeting policies.

In Chapter Two we have listed some studies on the effect of fiscal policy on inflation rate. As oppose to these studies –each one based on different theoretical assumptions, in that study, we covered all the theories on price level determination. Moreover, based on the FTPL argument that the government may choose inflation tax even with low dependence on money seignorage, we did our analyses for U.S inflation rate. Hence

another further study would be to analyse the inflation rate of country with high government debt.

APPENDIX A. MATHEMATICAL SOLUTIONS FOR THE PRICE LEVEL

DETERMINATION

Appendix A solves the model given in Chapter Two for the cases of

1. Unpleasant Monetarist Arithmetic: Ricardian policy; active fiscal and passive

monetary policy

2. Fiscal Theory of Price Level: Non Ricardian policy; active fiscal and active

monetary policy

and analyses the long run effect of a fiscal shock on price level in each of these cases.

In the case of an active fiscal policy the fiscal authority will set his policy variables

$\{g_t\}$, $\{tx_t\}$ and $\{b_t\}$ independent of the solvency of the government budget constraint. I

will assume that they are set as constants. Assuming a constant output level, it follows

that the consumption level must also be constant. So, the first order conditions of the

household optimisation problem imply:

$$\frac{1}{R_t} = \beta \frac{p_t}{p_{t+1}} \tag{5}$$

$$m_t = c\left(\frac{R_t}{R_t - 1}\right) \tag{6}$$

where c is the constant consumption level.

Note that under the aggregate resource constraint, equation (9), the household and

government budget constraints are represented by the same equation:

$$p_t(g_t - tx_t) = M_t - M_{t-1} + B_t - (R_{t-1}) B_{t-1}$$

Unpleasant Monetarist Arithmetic

Ricardian policy: active fiscal and passive monetary policy

Under a Ricardian environment, an active fiscal policy leaves the monetary authority with the burden of the solvency of the government budget constraint. Hence, the monetary policy is driven by the fiscal policy variables.

For a constant level of primary surpluses, $D = (g - tx_t)$ and constant real valued government debt, b, the government's flow budget constraint is[75]:

$$D = m_t - m_{t-1}\left(\frac{p_{t-1}}{p_t}\right) + b - R_{t-1}\left(\frac{p_{t-1}}{p_t}\right)b \qquad (8)$$

Given fiscal policy and the demand conditions -equation (5) and (6)- equation (8) becomes a function of R, D and b.

$$D = c\left(\frac{R_t}{R_t - 1}\right) - c\left(\frac{R_{t-1}}{R_{t-1} - 1}\right)\left(\frac{1}{\beta R_{t-1}}\right) + b\left(1 - R_{t-1}\left(\frac{1}{\beta R_{t-1}}\right)\right) \qquad (14)$$

Equation (14) is solved for the path of the monetary policy variable, the interest rate, as a function of D and b:

$$R_t = 1 + \frac{R_{t-1} - 1}{(R_{t-1} - 1)\left(\frac{D}{c} - \frac{b}{c}\left(1 - \frac{1}{\beta}\right) - 1\right) + \frac{1}{\beta}} \qquad (15)$$

Then the rate of inflation, $\pi_t = \frac{p_t}{p_{t-1}}$ follows from (5) and (15):

$$\pi_t = \beta + \frac{\beta(R_{t-2} - 1)}{(R_{t-2} - 1)\left(\frac{D}{c} - \frac{b}{c}\left(1 - \frac{1}{\beta}\right) - 1\right) + \frac{1}{\beta}} \qquad (17)$$

Suppose, $\phi = \frac{D - b(1 - \alpha) - c}{c}$ and $\alpha = \frac{1}{\beta}$. Thus, $R_t = 1 + \frac{R_{t-1} - 1}{(R_{t-1} - 1)\phi + \alpha}$

The steady state interest rates must, from (15), solve:

$$(R\phi - \phi + \alpha)(R - 1) = (R - 1)$$

---

[75] Under the aggregate resource constraint: $y_t = c_t + g_t$ the household and government budget constraints are same.
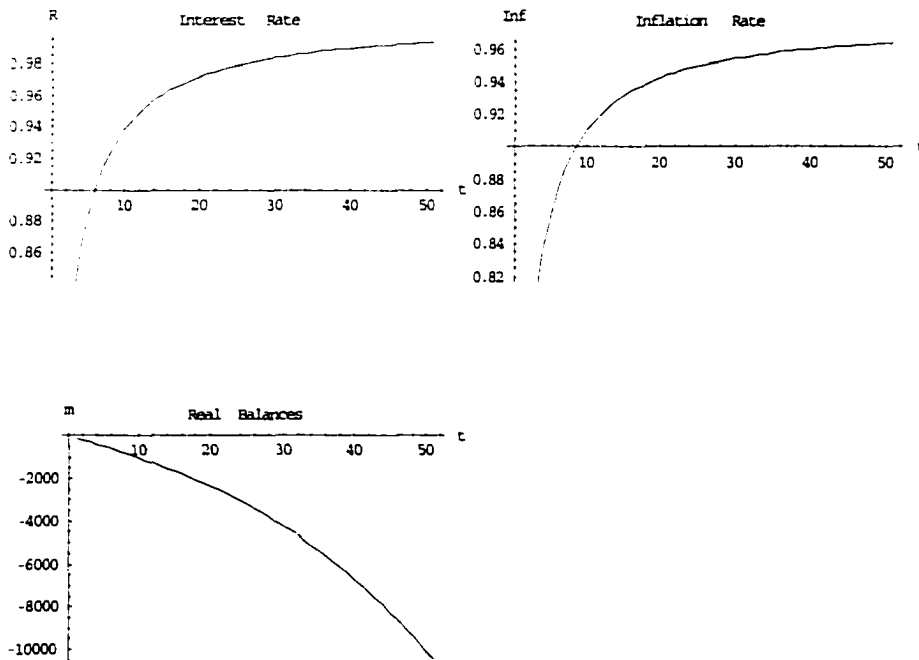
There are two steady state values of interest rate:

$$R_1 = 1 \quad \text{and} \quad R_2 = \frac{1 + \phi - \alpha}{\phi}$$

To check for the stability of the steady state, $\dfrac{dR_t}{dR_{t-1}} = \dfrac{\alpha}{(\phi R_{t-1} + \alpha - \phi)^2} > 0$

$$\left. \frac{dR_t}{dR_{t-1}} \right|_{R=R_1} = \frac{1}{\alpha} = \beta < 1 \quad \text{and} \quad \left. \frac{dR_t}{dR_{t-1}} \right|_{R=R_2} = \alpha = \frac{1}{\beta} > 1$$

Hence. $R_1 = 1$ is a stable and R_2 is an unstable equilibrium.

Our simulation results for unpleasant monetarist arithmetic shows that for $\beta = 0.97$, interest rate. inflation rate and real balances evolve as:

Note that, according to monetarist theory, inflation rate determined by the monetary authority. Therefore, given monetary variables, the government debt evolves as:
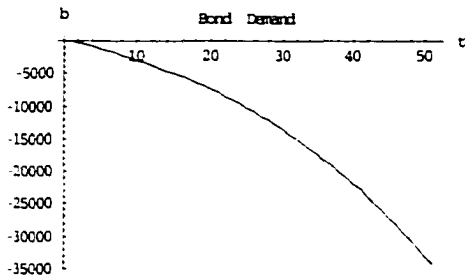
Fiscal Theory of the Price Level

Non Ricardian policy: active fiscal and active monetary policy

As opposed to unpleasant monetarist argument the fiscal theory of the price level argues that the price level is determined by the fiscal policy, even if the monetary authority follows an active policy.

In a non Ricardian environment, an active fiscal policy requires that the households will hold government bonds. However, if households know that the government will roll over its debt without ever retiring it, they will not demand any bonds. The condition to prevent zero demand is for the households to know that the government will finance its future debt with future surplus. Therefore, in order to prevent ponzi games the transversality condition has to hold. Therefore, the FTPL works with the intertemporal government budget constraint.

The central bank sets the rate of interest such that:

$$R_t = \frac{1}{\beta} \frac{p_{t+1}}{p_t} \qquad (5')$$

The money supply is:

$$\frac{M_t}{p_t} = c\left(\frac{p_{t+1}}{p_{t+1} - \beta p_t}\right) \tag{6'}$$

Recall that the government budget constraint together with the transversality

condition derives the intertemporal budget constraint:

$$\frac{p_{t-1}}{p_t}\left(\frac{M_{t-1}}{p_{t-1}} + (R_{t-1})b\right) = \sum_{j=0}^{\infty}\left(\prod_{s=t}^{t+j-1}\frac{1}{R_s}\frac{p_{s+1}}{p_s}\right)\left(\frac{R_{t+j}-1}{R_{t+j}}\frac{M_{t+j}}{p_{t+j}} - D_{t+j}\right) \tag{19}$$

Let $\Omega = \dfrac{p_{t-1}}{p_t}\left(\dfrac{M_{t-1}}{p_{t-1}} + R_{t-1}b\right)$, $\Gamma = \displaystyle\prod_{s=t}^{t-1}\dfrac{1}{R_s}\dfrac{p_{s+1}}{p_s}\left(\dfrac{R_t-1}{R_t}\dfrac{M_t}{p_t} - D_t\right)$ and

$\Psi = \displaystyle\sum_{j=1}^{\infty}\left(\prod_{s=t}^{t+j-1}\frac{1}{R_s}\frac{p_{s+1}}{p_s}\right)\left(\frac{R_{t+j}-1}{R_{t+j}}\frac{M_{t+j}}{p_{t+j}} - D_{t+j}\right)$. Thus. equation (19) is: $\Omega = \Gamma + \Psi$

Given the fiscal policy $D_t = D_{t+1} = D$ and equations (5') and (6');

$$\Omega = \frac{cp_t}{p_t - \beta p_{t-1}} + \frac{b}{\beta}, \quad \Gamma = c - D \text{ and } \Psi = \sum_{j=1}^{\infty}\beta^j(c-D) \tag{20}$$

Thus equation (19) reduces to:

$$\frac{cp_t}{p_t - \beta p_{t-1}} + \frac{b}{\beta} = c - D + \sum_{j=1}^{\infty}\beta^j(c-D)$$

The price level evolves according to:

$$p_t = p_{t-1}\left(\frac{\displaystyle\sum_{j=0}^{\infty}\beta^{j+1}(c-D) - b}{\displaystyle\sum_{j=0}^{\infty}\beta^j(c-D) - \frac{b}{\beta} - c}\right) \tag{21}$$
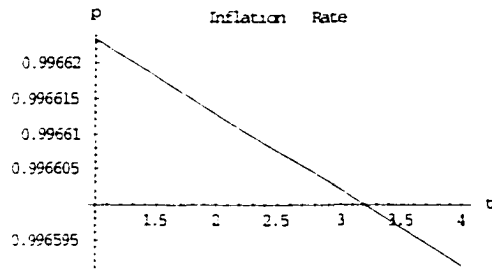
and the rate of inflation is:

$$\pi_t = \left(\frac{-b + (c - D) \sum_{j=0}^{\infty} \beta^{j+1}}{-\frac{b}{\beta} - c + (c - D) \sum_{j=0}^{\infty} \beta^j} \right) \tag{22}$$

A positive fiscal shock (decrease in taxes) increases inflation rate:

$$\frac{d\pi}{dD} = \frac{c \sum_{j=0}^{\infty} \beta^{j+1}}{\left(-\frac{b}{\beta} - c + (c - D) \sum_{j=0}^{\infty} \beta^j \right)^2}$$

Our simulation results for $\beta = 0.97$, $R = 0.7$ also indicate the negative relation

between taxes and inflation rate:

APPENDIX B. EXAMPLES OF VECTOR AUTOREGRESSION MODEL

IDENTIFICATION RESTRICTIONS

Assume the researcher defines a dynamic system of equations for the variables, $\{k_t\}$, $\{w_t\}$ and $\{q_t\}$ as a first order vector autoregression process. The structural form of the system is:

$$k_t = -B_{12} w_t - B_{13} q_t + D_{11}(1) k_{t-1} + D_{12}(1) w_{t-1} + D_{13}(1) q_{t-1} + \varepsilon_{k,t}$$

$$w_t = -B_{21} k_t - B_{23} q_t + D_{21}(1) k_{t-1} + D_{22}(1) w_{t-1} + D_{23}(1) q_{t-1} + \varepsilon_{w,t}$$

$$q_t = -B_{31} k_t - B_{32} w_t + D_{31}(1) k_{t-1} + D_{32}(1) w_{t-1} + D_{33}(1) q_{t-1} + \varepsilon_{q,t}$$

where these series satisfies all the properties given in Chapter Five.

The matrix form is:

$$\begin{bmatrix} 1 & z_{12} & z_{13} \\ z_{21} & 1 & z_{23} \\ z_{31} & z_{32} & 1 \end{bmatrix} \begin{bmatrix} k_t \\ w_t \\ q_t \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \begin{bmatrix} k_{t-1} \\ w_{t-1} \\ q_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{kt} \\ \varepsilon_{wt} \\ \varepsilon_{qt} \end{bmatrix}$$

For estimation purposes the researcher should use the VAR in standard form:

$$\begin{bmatrix} k_t \\ w_t \\ q_t \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} k_{t-1} \\ w_{t-1} \\ q_{t-1} \end{bmatrix} + \begin{bmatrix} e_{kt} \\ e_{wt} \\ e_{qt} \end{bmatrix}$$

and for innovation accounting he/she will need the system in VMA form:

$$\begin{bmatrix} k_t \\ w_t \\ q_t \end{bmatrix} = \begin{bmatrix} C_{11}(L) & C_{12}(L) & C_{13}(L) \\ C_{21}(L) & C_{22}(L) & C_{23}(L) \\ C_{31}(L) & C_{32}(L) & C_{33}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix}$$

The researcher may use different methods to impose the restrictions to identify the structural VAR.

Sims Methodology: The researcher may apply the Choleski decomposition and restrict the upper triangle of the B matrix for the given ordering of the variables; k_t, w_t, q_t. Thus, the restrictions: $z_{12} = 0$, $z_{13} = 0$ and $z_{23} = 0$, define the model as:

$$
\begin{bmatrix} 1 & 0 & 0 \\ z_{21} & 1 & 0 \\ z_{31} & z_{32} & 1 \end{bmatrix}
\begin{bmatrix} k_t \\ w_t \\ q_t \end{bmatrix}
=
\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}
\begin{bmatrix} k_{t-1} \\ w_{t-1} \\ q_{t-1} \end{bmatrix}
+
\begin{bmatrix} \varepsilon_{k,t} \\ \varepsilon_{w,t} \\ \varepsilon_{q,t} \end{bmatrix}
$$

Sims-Bernanke Methodology: It is possible that the economic theory already suggests specific relations among variables. Suppose the theory suggests that k_t has no contemporaneous effect on w_t and q_t. Moreover, q_t has no contemporaneous effect on k_t. That is, $z_{13} = 0$, $z_{21} = 0$ and $z_{31} = 0$. Hence, the system is defined by the theory as:

$$
\begin{bmatrix} 1 & z_{12} & 0 \\ 0 & 1 & z_{23} \\ 0 & z_{32} & 1 \end{bmatrix}
\begin{bmatrix} k_t \\ w_t \\ q_t \end{bmatrix}
=
\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}
\begin{bmatrix} k_{t-1} \\ w_{t-1} \\ q_{t-1} \end{bmatrix}
+
\begin{bmatrix} \varepsilon_{k,t} \\ \varepsilon_{w,t} \\ \varepsilon_{q,t} \end{bmatrix}
$$

Blanchard-Quah Decomposition: If the theory imposes restrictions on the long run and short run behaviour of the variables, Blanchard-Quah decomposition uses these restrictions to differentiate the temporary and permanent effects of various shocks on variables. Lets assume that researcher works on a bivariate system of k_t and w_t, which are effected by the shocks ε_1 and ε_2. It is also true that k_t is not stationary but the first difference of k_t, Δk_t, and w_t is.[76] Suppose the theory predicts that ε_2 does not have a long

---

[76] In their paper Blanchard-Quah works with the real GNP and the unemployment rate. They argue that it is possible to differentiate the temporary and permanent components of the real GNP using the theoretical argument that the aggregate demand shocks have no long run effect on the long run aggregate supply.

run effect on Δk_t sequence. Therefore, the following system is identified by the
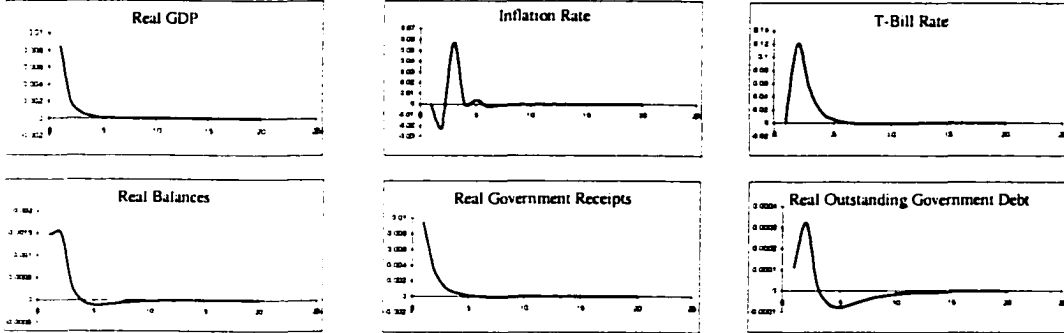
restriction, $\sum_{i=0}^{\infty} c_{i,12} = 0$:

$$\begin{bmatrix} \Delta k_t \\ w_t \end{bmatrix} = \begin{bmatrix} C_{11}(L) & C_{12}(L) \\ C_{21}(L) & C_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$
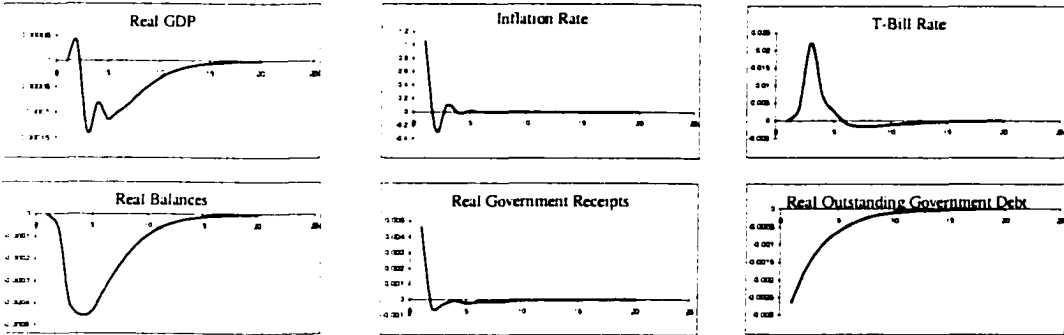
APPENDIX C. STRUCTURAL VECTOR AUTOREGRESSION MODEL RESULTS
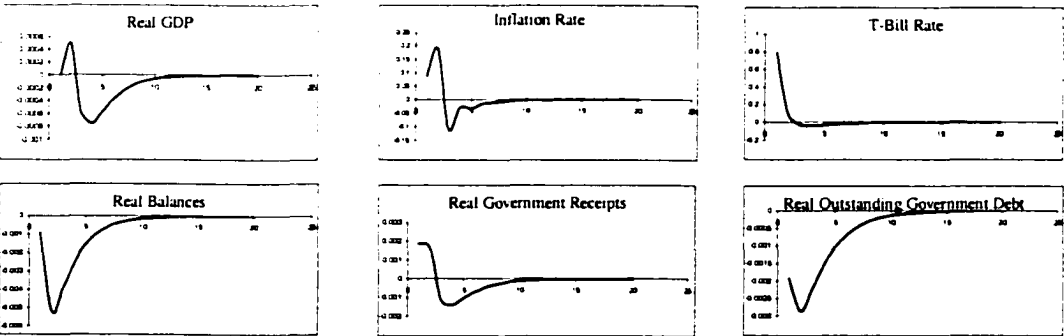
MONETARIST THEORY

Responses to Supply Shock

Responses to Demand Shock

Responses to Monetary Policy Shock

Responses to Money Demand Shock

Real GDP

Inflation Rate

T-Bill Rate

Real Balances

Real Government Receipts

Real Outstanding Government Debt

Responses to Fiscal Policy Shock

Real GDP

Inflation Rate

T-Bill Rate

Real Balances

Real Government Receipts

Real Outstanding Government Debt

Responses to Bond Demand Shock

Real GDP

Inflation Rate

T-Bill Rate

Real Balances

Real Government Receipts

Real Outstanding Government Debt

UNPLEASANT MONETARIST ARITHMETIC

Responses to Supply Shock

Real GDP

Inflation Rate

T-Bill Rate

Real Balances

Real Government Receipts

Real Outstanding Government Debt

Responses to Demand Shock

Real GDP

Inflation Rate

T-Bill Rate

Real Balances

Real Government Receipts

Real Outstanding Government Debt

Responses to Monetary Policy Shock

Real GDP

Inflation Rate

T-Bill Rate

Real Balances

Real Government Receipts

Real Outstanding Government Debt

Responses to Money Demand Shock

| Real GDP | Inflation Rate | T-Bill Rate |

| Real Balances | Real Government Receipts | Real Outstanding Government Debt |

Responses to Fiscal Policy Shock

| Real GDP | Inflation Rate | T-Bill Rate |

| Real Balances | Real Government Receipts | Real Outstanding Government Debt |

Responses to Bond Demand Shock

| Real GDP | Inflation Rate | T-Bill Rate |

| Real Balances | Real Government Receipts | Real Outstanding Government Debt |

FISCAL THEORY OF PRICE LEVEL

Responses to Supply Shock
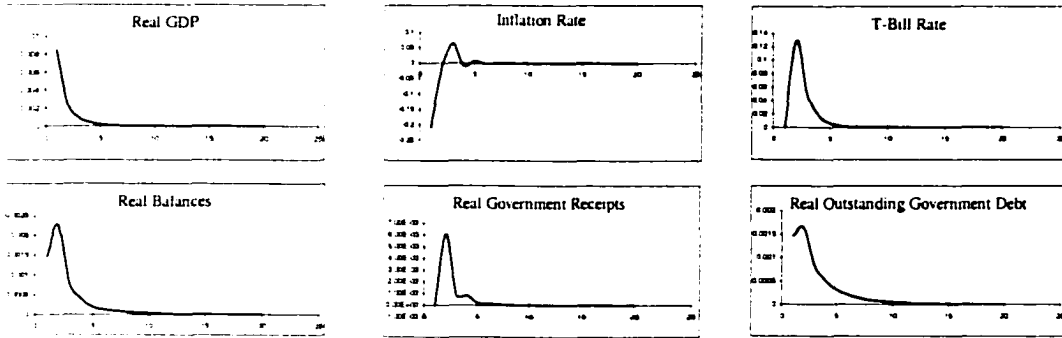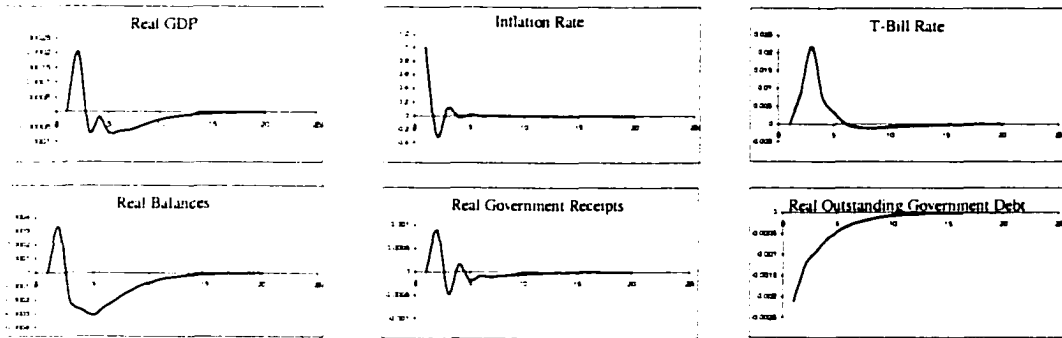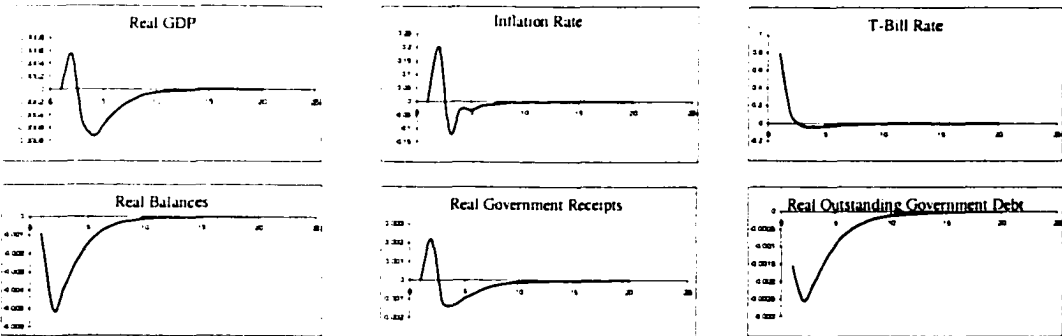
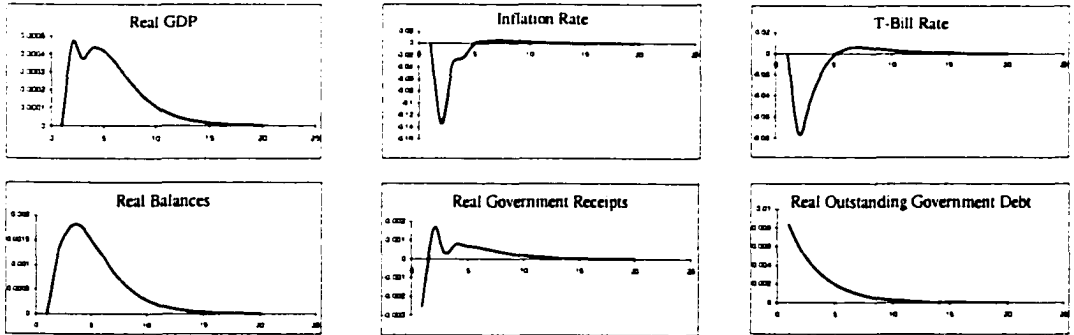**Responses to Demand Shock**

Responses to Monetary Policy Shock

Responses to Money Demand Shock

Real GDP

Inflation Rate

T-Bill Rate

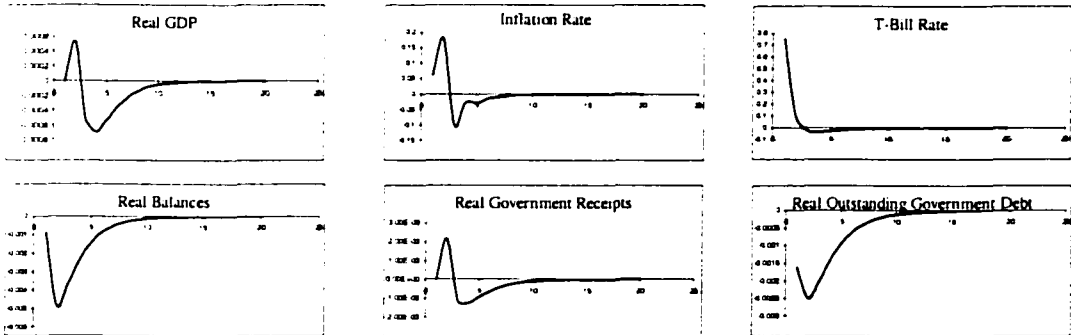Real Balances

Real Government Receipts

Real Outstanding Government Debt

Responses to Fiscal Policy Shock

Real GDP

Inflation Rate

T-Bill Rate

Real Balances

Real Government Receipts

Real Outstanding Government Debt

Responses to Bond Demand Shock

Real GDP

Inflation Rate

T-Bill Rate

Real Balances

Real Government Receipts

Real Outstanding Government Debt
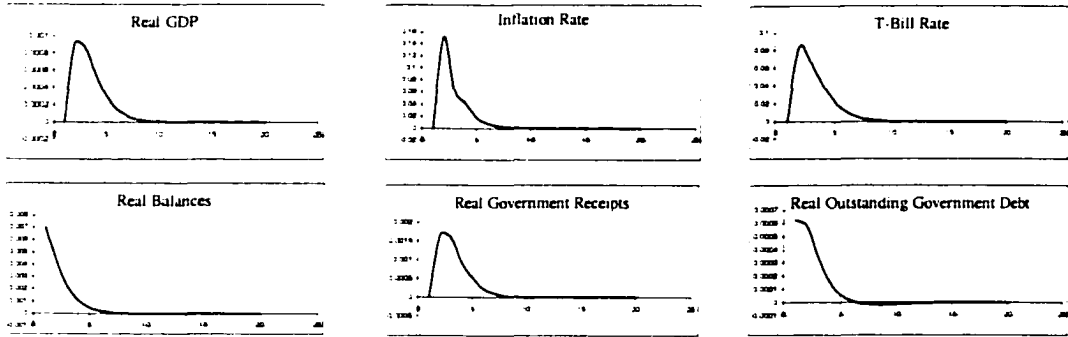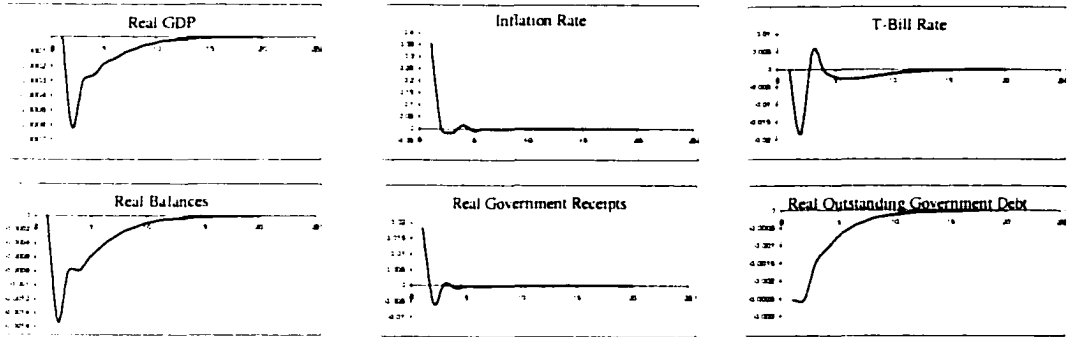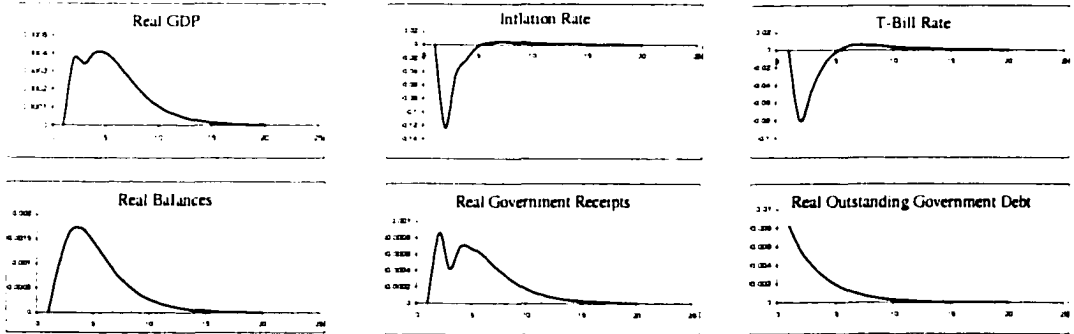
MONETARIST THEORY

Fraction of the forecast –error variance attributed to aggregate supply shock

Horizon	y	π	R	m	tx	b
1	100	0	0	4.1	26.6	0.01
3	98.1	0.3	2.5	3.6	26.8	0.08
5	94.1	0.3	2.6	3.1	26.4	0.08
7	93.9	0.3	2.6	3.1	26.3	0.08
10	93.9	0.3	2.6	3.1	26.3	0.08

Fraction of the forecast –error variance attributed to demand shock

Horizon	y	π	R	m	tx	b
1	0	99.3	0	0	6.4	8.6
3	0.03	91.4	0.07	0.1	5.7	8.01
5	0.07	91.0	0.08	0.5	5.6	7.9
7	0.08	91	0.08	0.6	5.6	7.9
10	0.08	91	0.08	0.6	5.6	7.9

Fraction of the forecast –error variance attributed to monetary policy shock

Horizon	Y	π	R	m	tx	b
1	0	0.7	100	1.6	1.1	4.5
3	0.3	4.2	1.7	34.2	2.0	11.2
5	2.0	4.3	2.01	36.4	2.9	12.3
7	2.1	4.3	2.01	36.3	2.9	12.4
10	2.1	4.3	2.01	36.3	2.9	12.4

Fraction of the forecast –error variance attributed to money demand shock

Horizon	y	π	R	m	tx	b
1	0	0	0	94.3	0.6	0.29
3	1.9	2.1	1.67	57.3	1.7	0.31
5	2.3	2.3	2.01	50.5	1.9	0.28
7	2.3	2.3	2.01	50.2	1.9	0.28
10	2.3	2.3	2.01	50.2	1.9	0.28

Fraction of the forecast –error variance attributed to fiscal policy shock

Horizon	y	π	R	m	tx	b
1	0	0	0	0	63.3	0
3	0.3	0.4	0.15	1.1	61.2	0.35
5	0.4	0.5	0.15	0.9	60.2	0.35
7	0.4	0.5	0.15	1.0	60.1	0.35
10	0.4	0.5	0.15	1.0	60.1	0.35

Fraction of the forecast –error variance attributed to bond demand shock

Horizon	y	π	R	m	tx	b
1	0	0	0	0	1.9	86.5
3	0.4	1.4	1.1	3.6	2.4	80
5	1.1	1.5	1.17	8.4	2.8	79
7	1.2	1.5	1.18	8.8	2.8	79
10	1.3	1.5	1.18	8.8	2.9	79

UNPLEASANT MONETARIST ARITHMETIC

Fraction of the forecast –error variance attributed to aggregate supply shock

Horizon	y	π	R	m	tx	b
1	100	3.6	3.6	3.2	0	1.62
3	96.5	3.3	6.9	2.8	10.2	1.8
5	93.9	3.3	6.8	2.43	10.12	1.69
7	93.8	3.3	6.8	2.41	10.11	1.69
10	93.8	3.3	6.8	2.41	10.11	1.69

Fraction of the forecast –error variance attributed to demand shock

Horizon	y	π	R	m	tx	b
1	0	85.5	0	0	0	5.2
3	0.05	80.9	0.08	0.12	0.23	4.4
5	0.06	80.6	0.09	0.28	0.24	4.3
7	0.07	80.6	0.09	0.3	0.25	4.3
10	0.07	80.6	0.09	0.3	0.25	4.3

Fraction of the forecast –error variance attributed to monetary policy shock

Horizon	y	π	R	m	tx	b
1	0	0.3	94.4	1.4	0	3.08
3	0.7	3.2	88.2	29.9	1.4	8.2
5	1.7	3.4	87.8	31.7	2.09	9.11
7	1.8	3.4	87.8	31.6	2.10	9.12
10	1.8	3.4	87.8	31.6	2.11	9.12

Fraction of the forecast –error variance attributed to money demand shock

Horizon	y	π	R	m	tx	b
1	0	0	0	95.3	0	0.46
3	1.9	1.9	1.07	58.7	1.28	0.53
5	2.3	2.08	2.09	51.7	1.56	0.48
7	2.4	2.08	2.09	51.4	1.55	0.48
10	2.4	2.08	2.09	51.3	1.55	0.48

Fraction of the forecast –error variance attributed to fiscal policy shock

Horizon	y	π	R	m	tx	b
1	0	10.5	1.9	0.03	100	8.8
3	0.5	9.3	1.7	5.8	86.6	11.9
5	0.83	9.29	1.8	6.1	85.4	12.1
7	0.84	9.29	1.8	6.23	85.3	12.1
10	0.85	9.29	1.8	6.24	85.3	12.1

Fraction of the forecast –error variance attributed to bond demand shock

Horizon	y	π	R	m	tx	b
1	0	0	0	0	0	80.6
3	0.3	1.21	1.2	3.09	0.2	72.9
5	0.9	1.24	1.30	7.6	0.5	72.2
7	1.02	1.24	1.31	7.96	0.62	72.1
10	1.03	1.24	1.31	7.99	0.63	72.1

FISCAL THEORY OF PRICE LEVEL

Fraction of the forecast −error variance attributed to aggregate supply shock

Horizon	y	π	R	m	tx	b
1	100	3.6	0	4.1	0	2.5
3	96.3	3.4	2.8	6.1	9.0	3.5
5	93.9	3.4	2.8	5.5	9.1	3.3
7	93.8	3.4	2.8	5.4	9.09	3.3
10	93.7	3.4	2.8	5.4	9.09	3.3

Fraction of the forecast −error variance attributed to demand shock

Horizon	y	π	R	m	tx	b
1	0	85.8	0	0	0	5.3
3	0.05	80.4	0.08	0.1	0.2	4.4
5	0.07	80.2	0.09	0.2	0.25	4.3
7	0.075	80.1	0.09	0.3	0.25	4.3
10	0.075	80.1	0.09	0.3	0.25	4.3

Fraction of the forecast −error variance attributed to monetary policy shock

Horizon	y	π	R	m	tx	b
1	0	0	100	94.3	0	2.8
3	0.7	3.8	94	56.8	1.4	8.3
5	1.9	3.98	93.6	33.6	2.2	9.2
7	1.9	3.99	93.6	33.5	2.2	9.2
10	1.9	3.99	93.6	33.4	2.2	9.2

Fraction of the forecast −error variance attributed to money demand shock

Horizon	y	π	R	m	tx	b
1	0	0	0	94.3	0	0.4
3	1.9	1.9	1.7	56.8	1.2	0.5
5	2.4	2.06	2.05	50.3	1.5	0.4
7	2.4	2.06	2.05	50.08	1.5	0.4
10	2.4	2.06	2.05	50.05	1.5	0.4

Fraction of the forecast −error variance attributed to fiscal policy shock

Horizon	y	π	R	m	tx	b
1	0	10.5	0	0	100	7.5
3	0.5	9.1	0.05	2.2	87.6	9.5
5	0.73	9.1	0.05	2.8	86.3	9.6
7	0.74	9.1	0.05	2.8	86.2	9.6
10	0.74	9.1	005	2.8	86.2	9.6

Fraction of the forecast −error variance attributed to bond demand shock

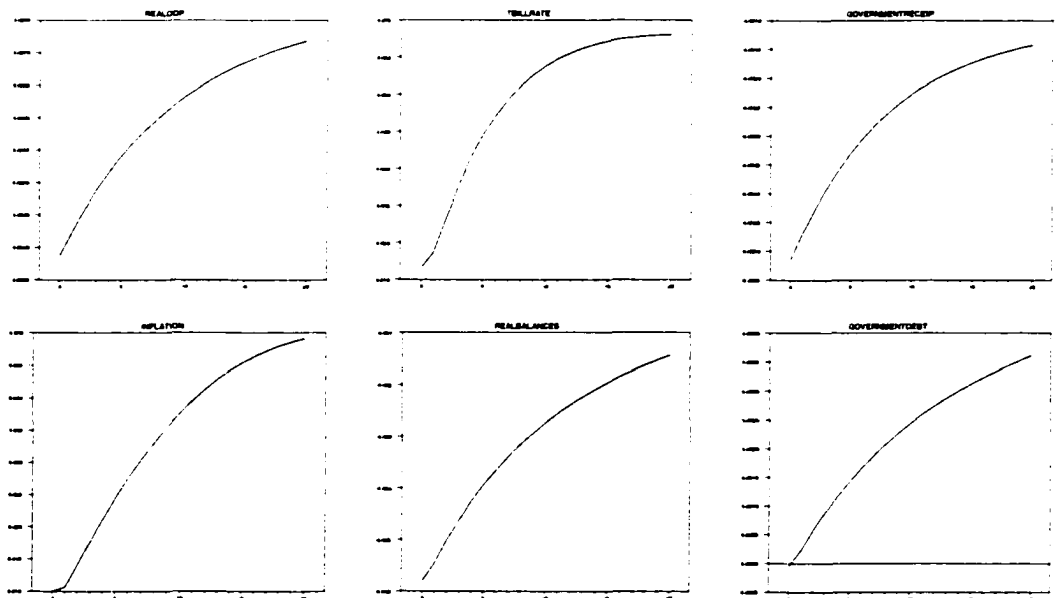| Horizon	y	π	R	m	tx	b
1	0	0	0	0	0	81.1
3	0.3	1.2	1.2	2.9	0.2	73.5
5	0.9	1.2	1.28	7.3	0.5	72.8
7	1.01	1.2	1.29	7.7	0.6	72.8
10	1.03	1.2	1.29	7.7	0.6	72.8

APPENDIX D. STRUCTURAL VECTOR ERROR CORRECTION MODEL RESULTS
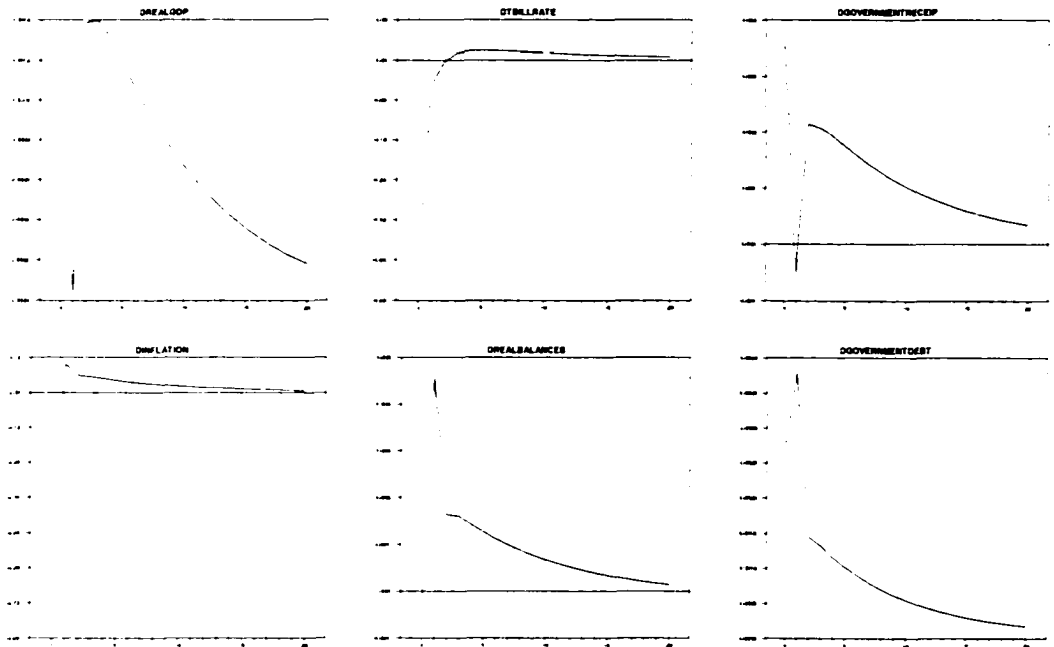
MONETARIST THEORY

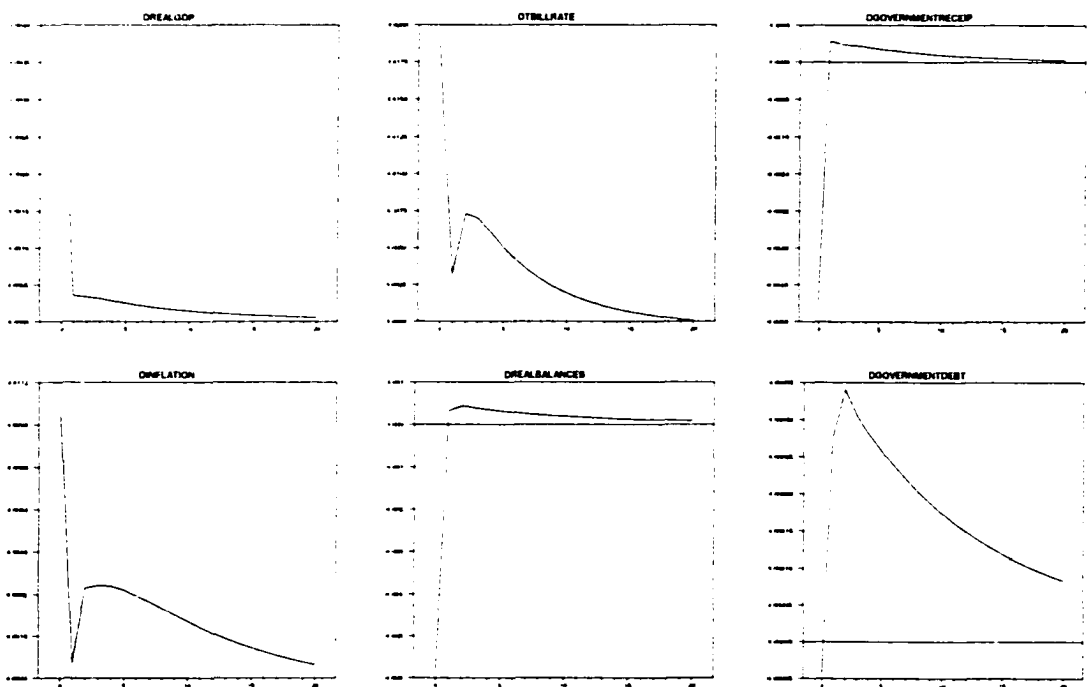- Impulse responses of variables in levels to technology shock
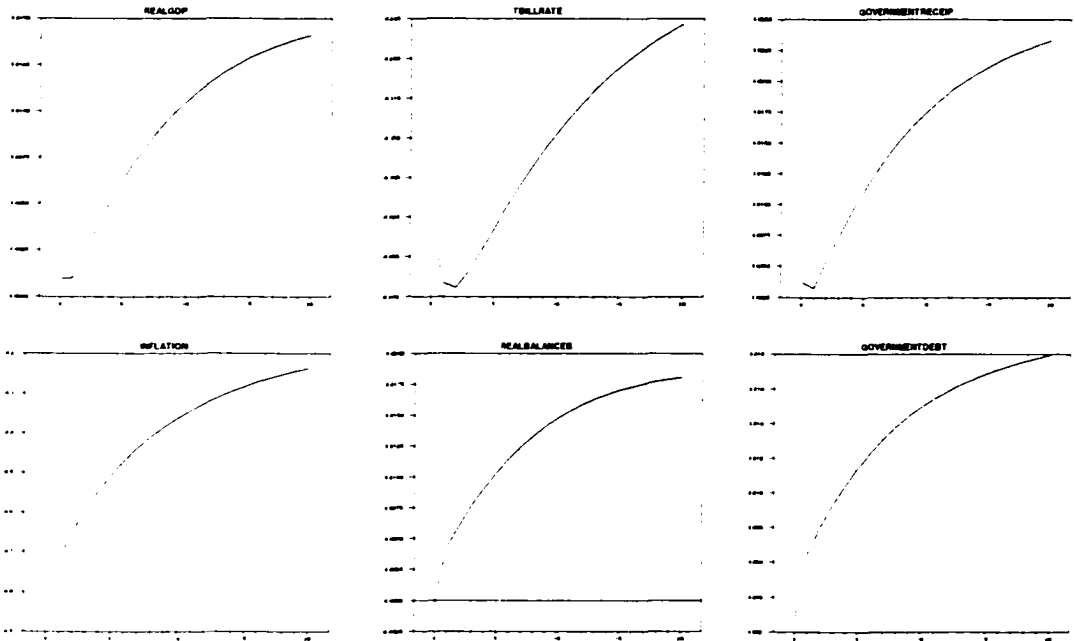
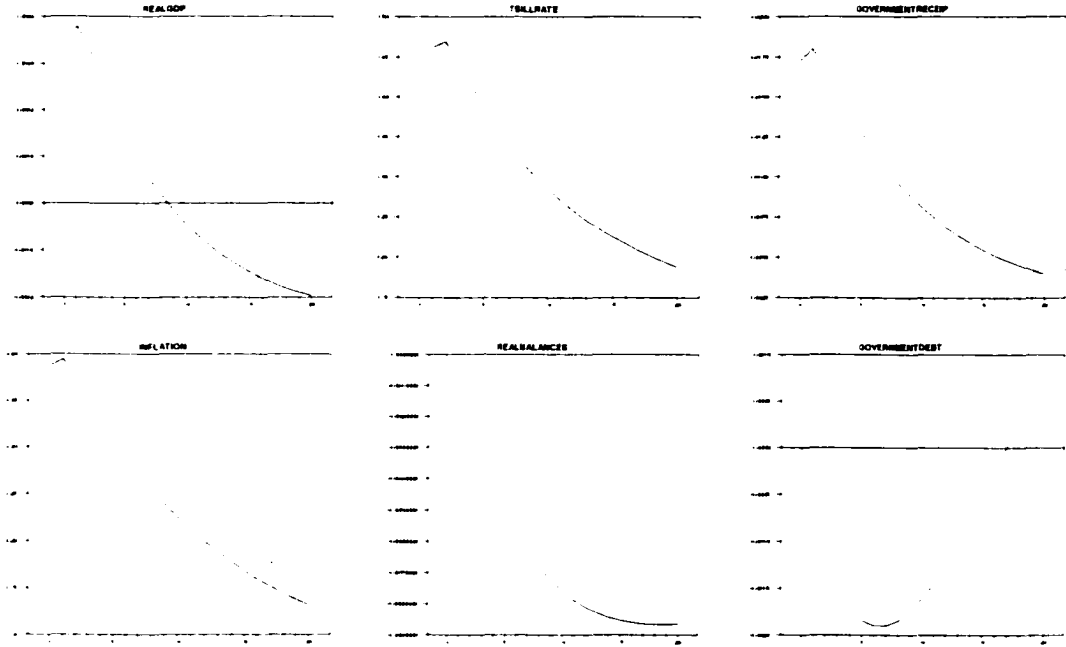- Impulse responses of variables in levels to monetary policy shock

- Impulse responses of variables in differences to technology shock
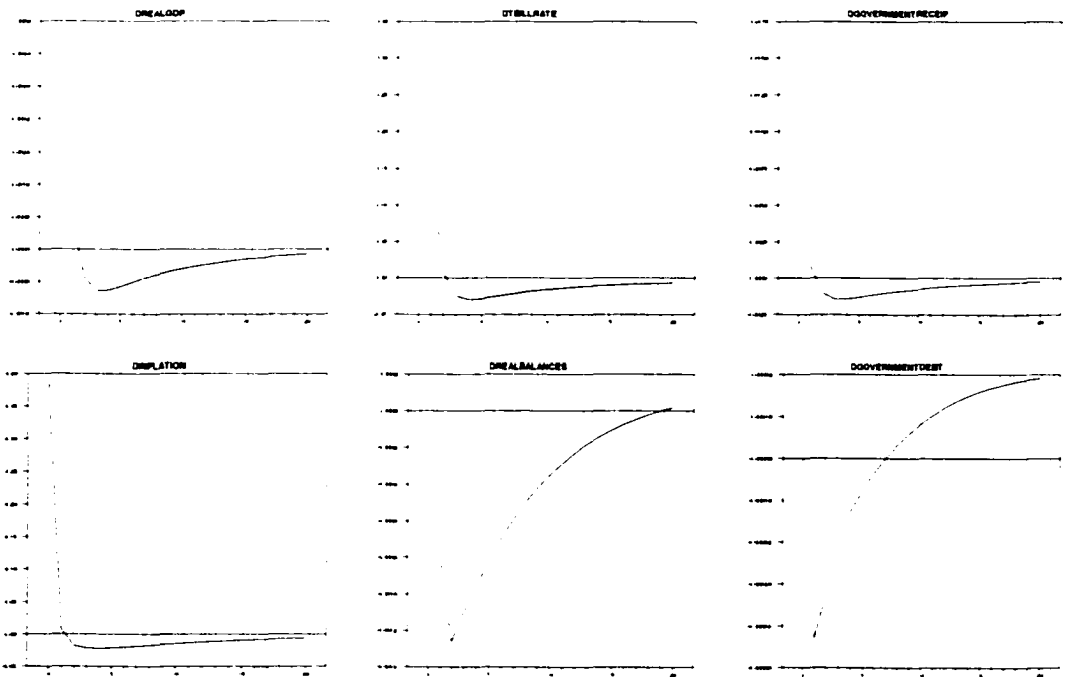

- Impulse responses of variables in differences to monetary policy shock

- Impulse responses of variables in levels to fiscal policy shock
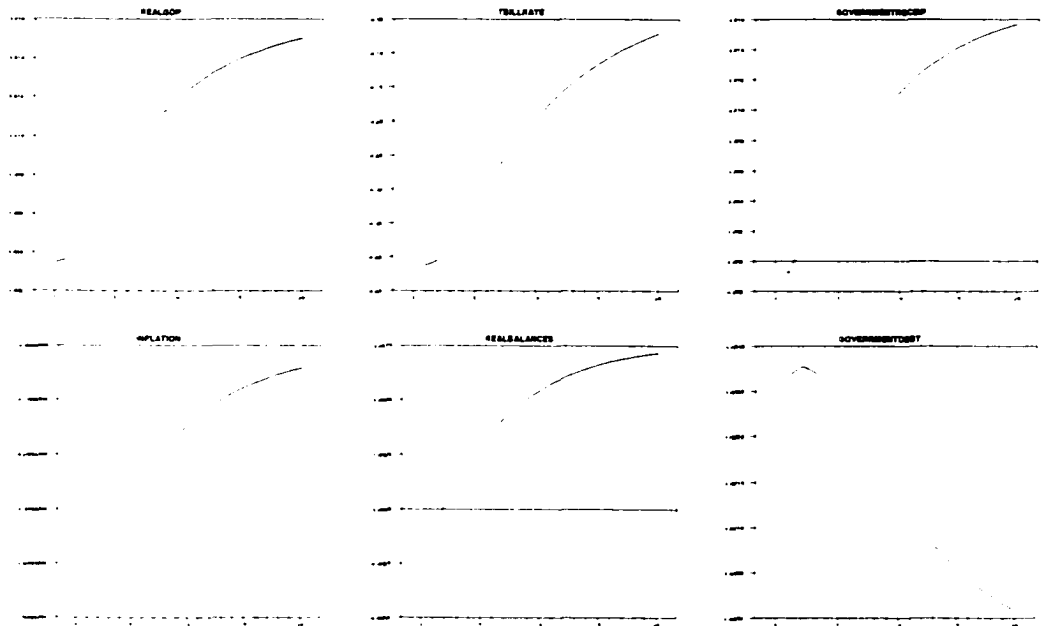

- Impulse responses of variables in differences to fiscal policy shock

FISCAL THEORY OF PRICE LEVEL

- Impulse responses of variables in levels to technology shock

- Impulse responses of variables in levels to monetary policy shock

- Impulse responses of variables in levels to fiscal policy shock

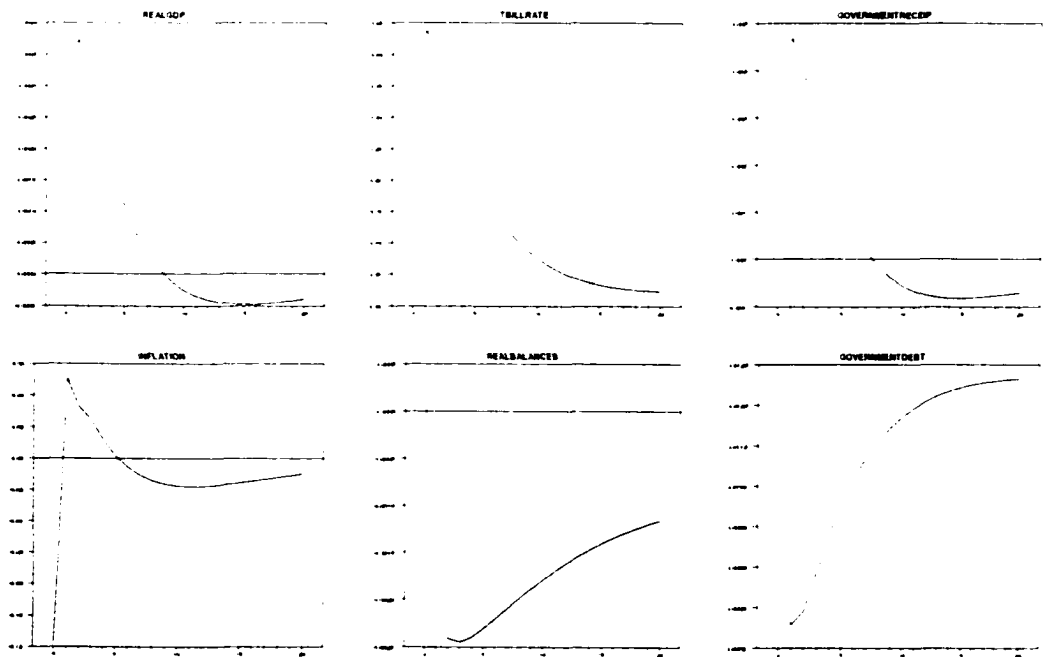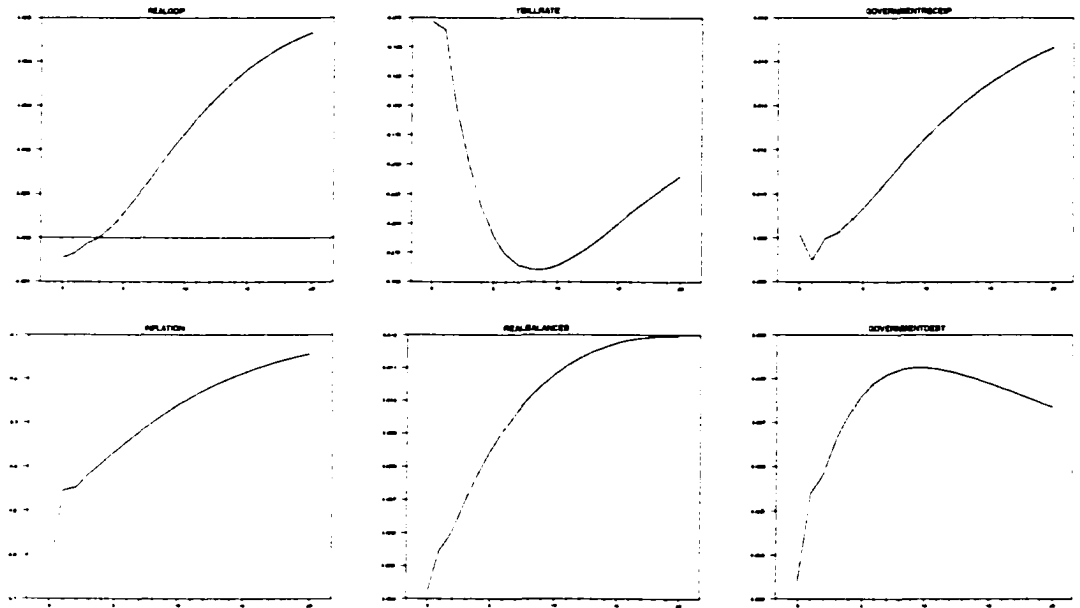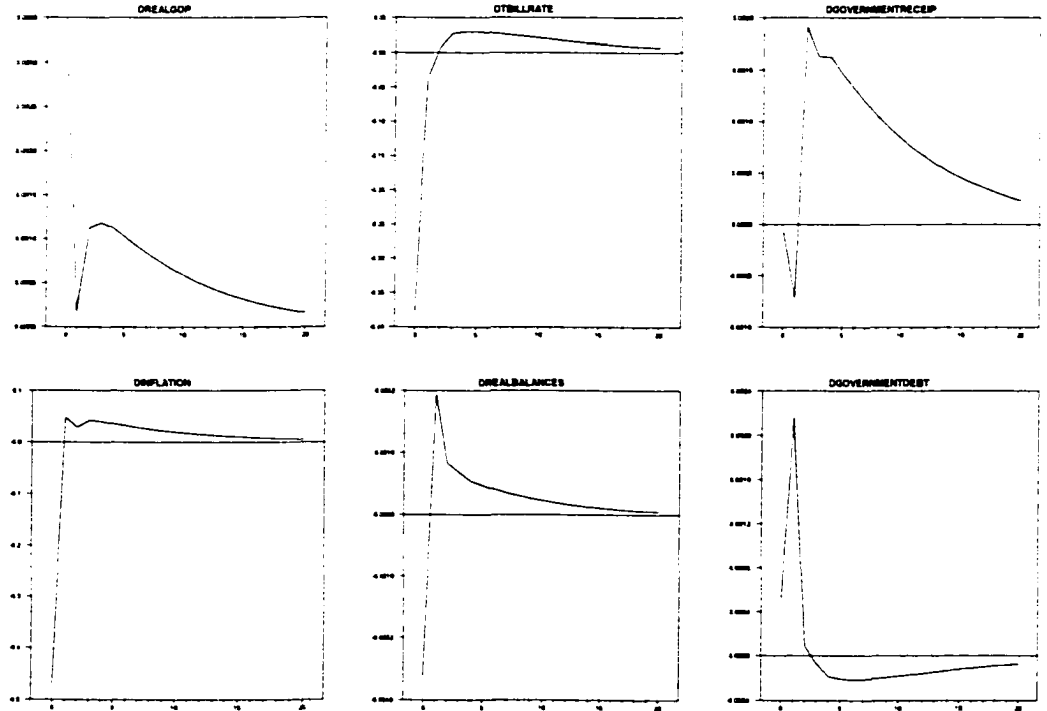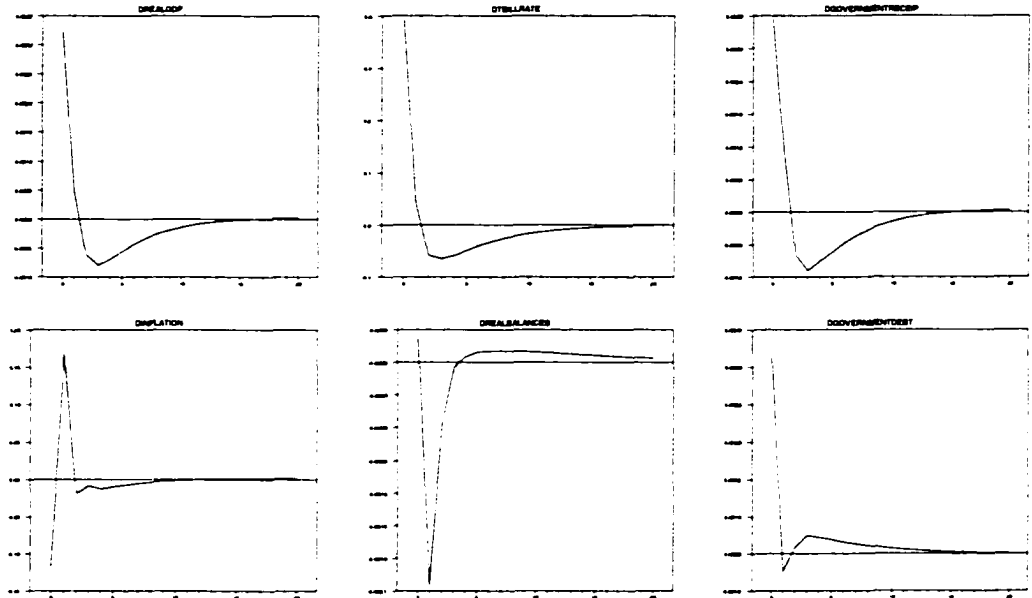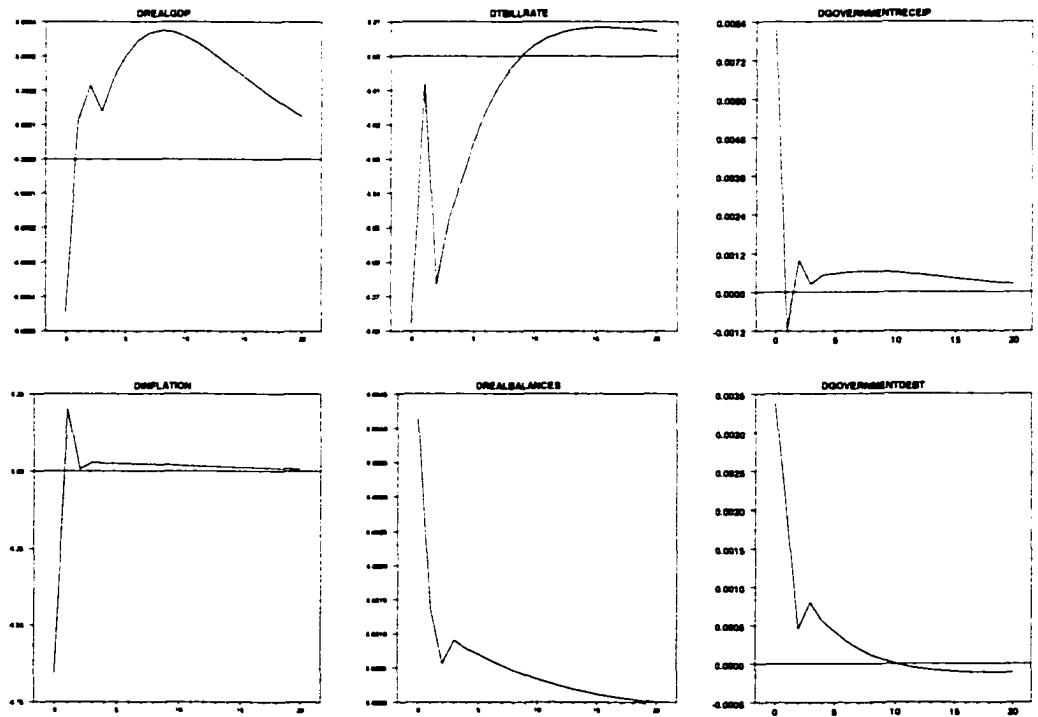- Impulse responses of variables in differences to technology shock

- Impulse responses of variables in differences to monetary policy shock

- Impulse responses of variables in differences to fiscal policy shock

MONETARIST THEORY

Fraction of the forecast –error variance attributed to technology shock

Horizon	y	π	R	m	tx	b
1	1.4	53.7	30.0	0.0	3.9	6.7
3	3.6	55.8	34.7	14.5	4.9	25.4
5	13.1	57.7	38.2	26.4	11.6	35.1
7	25.5	58.9	40.7	36.0	20.8	42.2
10	41.1	60.1	43.1	46.2	35.7	49.7

Fraction of the forecast –error variance attributed to monetary policy shock

Horizon	y	π	R	m	tx	b
1	24.0	0.0	0.1	35.4	7.6	0.0
3	25.7	0.0	0.1	23.1	6.3	0.1
5	28.9	0.0	0.2	17.2	5.7	0.3
7	30.1	0.1	0.3	13.0	5.0	0.5
10	29.0	0.1	0.5	8.9	4.1	0.9

Fraction of the forecast –error variance attributed to joint effect of permanent shocks

Horizon	y	π	R	m	tx	b
1	25.4	53.7	30.0	35.4	11.6	6.7
3	29.3	55.8	34.7	37.7	11.3	25.5
5	42.1	57.7	38.4	43.6	17.2	35.4
7	55.5	58.9	41.0	49.0	25.9	42.7
10	70.1	60.2	43.6	55.1	39.8	50.6

Fraction of the forecast –error variance attributed to joint effect of temporary shocks

Horizon	y	π	R	m	tx	b
1	74.6	46.3	70.0	64.6	88.4	93.3
3	70.7	44.2	65.3	62.3	88.7	74.5
5	57.9	42.3	61.6	56.4	82.8	64.6
7	44.5	41.1	59.0	51.0	74.1	57.3
10	29.9	39.8	56.4	44.9	60.2	49.4

Fraction of the forecast –error variance attributed to fiscal policy shock

Horizon	y	π	R	m	tx	b
1	45.3	12.2	16.6	0.0	86.6	0.0
3	0.1	3.5	0.4	12.9	1.9	73.3
5	34.4	17.5	32.4	4.6	80.9	0.8
7	24.1	18.6	33.9	7.6	72.4	1.0
10	14.1	19.7	34.7	11.1	58.7	1.0

FISCAL THEORY OF PRICE LEVEL

Fraction of the forecast –error variance attributed to technology shock

Horizon	y	π	R	m	tx	b
1	19.6	19.7	23.9	17.7	0.0	0.2
3	24.9	24.2	24.6	5.3	0.2	2.9
5	40.3	24.3	25.9	3.8	2.2	3.0
7	54.8	23.7	26.2	4.4	6.5	2.6
10	68.6	22.7	25.5	6.1	14.7	2.1

Fraction of the forecast –error variance attributed to monetary policy shock

Horizon	y	π	R	m	tx	b
1	17.1	1.2	26.3	0.1	3.9	48.2
3	17.6	0.8	25.4	2.6	6.0	36.5
5	12.7	0.6	23.1	3.1	4.8	37.1
7	8.5	0.5	20.7	2.8	3.6	39.2
10	4.8	0.4	17.9	2.3	2.5	42.2

Fraction of the forecast –error variance attributed to fiscal policy shock

Horizon	y	π	R	m	tx	b
1	0.3	38.4	1.0	17.3	20.2	7.8
3	0.2	36.5	1.9	23.8	21.4	14.1
5	0.1	36.6	4.6	28.2	24.2	17.6
7	0.3	37.1	7.9	31.7	29.2	20.0
10	1.1	37.8	12.3	35.1	34.6	21.8

Fraction of the forecast –error variance attributed to joint effect of permanent shocks

Horizon	y	π	R	m	tx	b
1	37.0	59.3	51.2	35.0	24.1	56.1
3	42.6	61.5	51.9	31.7	27.6	53.4
5	53.1	61.5	53.6	35.0	32.2	57.7
7	63.5	61.3	54.8	38.9	29.3	61.8
10	74.5	60.9	55.7	43.6	51.8	66.1

Fraction of the forecast –error variance attributed to joint effect of temporary shocks

Horizon	y	π	R	m	tx	b
1	63.0	40.7	48.8	65.0	75.9	43.9
3	57.4	38.5	48.1	68.3	72.4	46.6
5	46.9	38.5	46.4	65.0	67.8	42.3
7	36.5	38.7	45.2	61.1	60.7	38.2
10	25.5	39.1	44.3	56.4	48.2	33.9

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